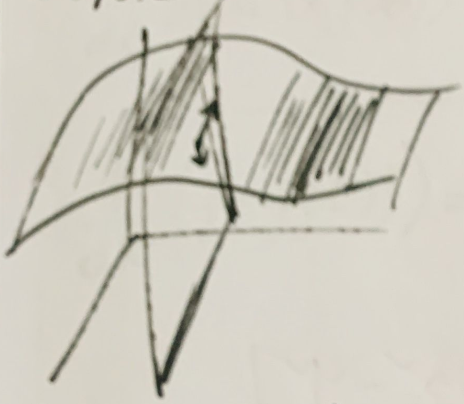
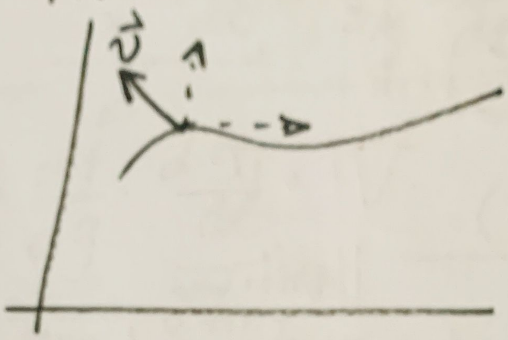
DIRECTIONAL DERIVATIVES:

Before...



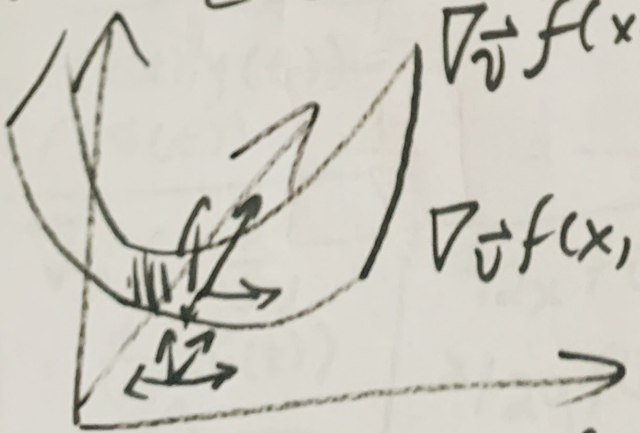
how much will a nudge parallel to x & y change the function?

Now...



how much will a nudge in $\vec{v}$ change the function?

If $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$, $\nabla_{\vec{v}} f(x,y) = a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y}$

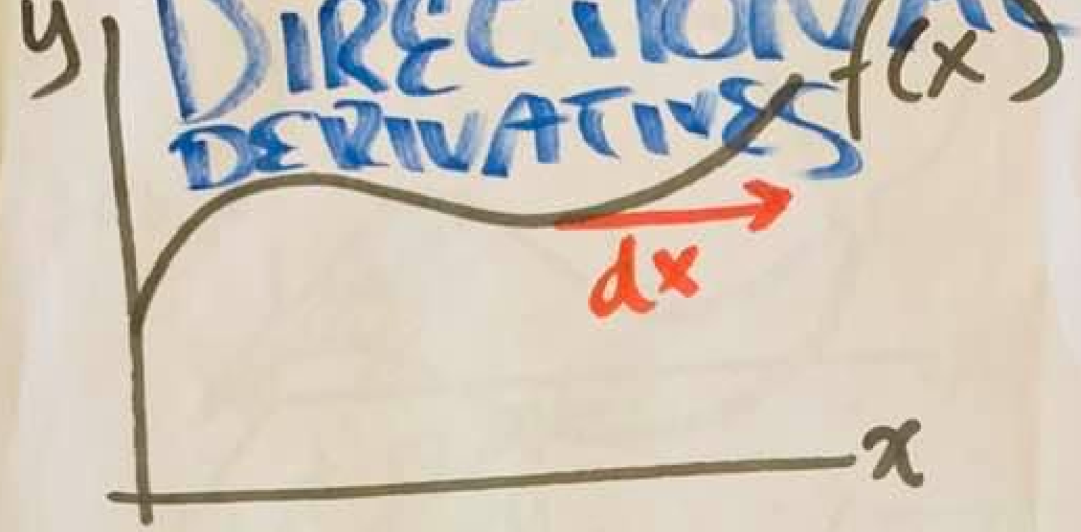


$$\nabla_{\vec{v}} f(x,y) = \begin{bmatrix} a \\ b \end{bmatrix} \circ \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix}$$

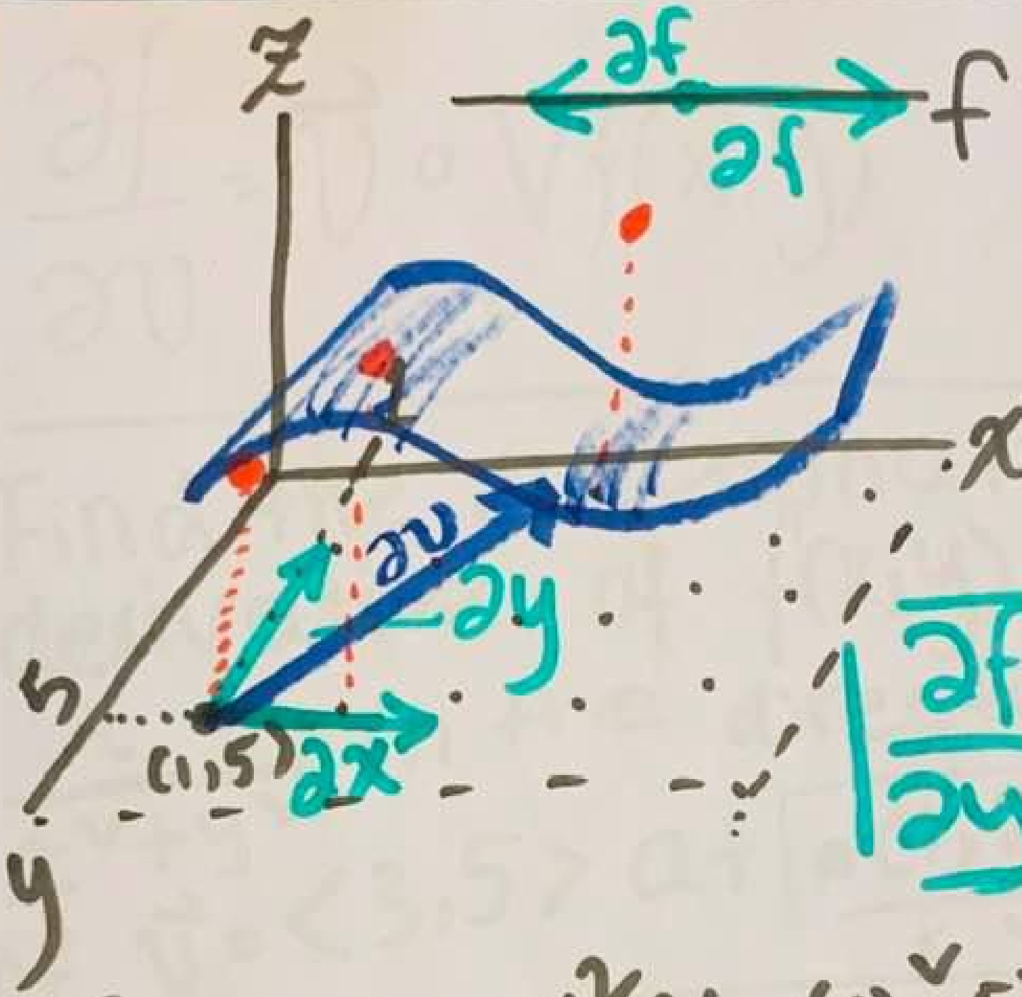
$$\nabla_{\vec{v}} f(x,y) = \vec{v} \circ \nabla f$$

$$D_{\vec{v}} f(x,y) = \lim_{h \to 0} \frac{f(x+ha, y+hb) - f(x,y)}{h}$$

DIRECTIONAL DERIVATIVES



$$\frac{df}{dx} = \frac{dy}{dx}$$



$$\left|\frac{\partial f}{\partial y}\right|$$

$$f(x,y) = x^2 y = (1)^2(5) = 5$$

$$\frac{\partial f(x,y)}{\partial x} \qquad \left|\frac{\partial f}{\partial x}\right|$$

$$\frac{\partial f}{\partial v} = \vec{v} \circ \nabla f(x,y)$$

Find the directional derivative of $f(x,y) = \dfrac{x}{x^2+y^2}$ in the direction of $\vec{v} = \langle 3,5 \rangle$ at $\boxed{\bullet(\underset{x}{1},\underset{y}{2})}$

$$\frac{\partial f}{\partial v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \circ \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$f(x,y) = \frac{x}{x^2+y^2}$$

$$\frac{\partial f}{\partial x} = \frac{(1)(x^2+y^2)-(2x)(x)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{0-(2y)(x)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \circ \begin{bmatrix} \frac{y^2-x^2}{(x^2+y^2)^2} \\ \frac{-2xy}{(x^2+y^2)^2} \end{bmatrix}$$

$$\frac{\partial f}{\partial v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \circ \begin{bmatrix} \frac{4-1}{25} = 3/25 \\ -4/25 \end{bmatrix}$$

$$\frac{\partial f}{\partial v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \circ \begin{bmatrix} 3/25 \\ -4/25 \end{bmatrix}$$

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<u>dot product</u>: $ac + bd = \begin{bmatrix} a \\ b \end{bmatrix} \circ \begin{bmatrix} c \\ d \end{bmatrix}$

$$(3)\left(\frac{3}{25}\right) + (5)\left(\frac{-4}{25}\right)$$

$$\nabla f = \frac{1}{25}\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\frac{9}{25} - \frac{20}{25} = \frac{-11}{25}$$

$$\frac{\partial f}{\partial v} = \frac{\vec{v} \circ \nabla f(x, y)}{|\vec{v}|} = \frac{\frac{-11}{25}}{\sqrt{34}}$$

$$\boxed{\frac{-11}{25\sqrt{34}}}$$

$$|\vec{v}| = \sqrt{3^2 + 5^2} = \sqrt{34}$$

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Find the directional derivative of $f(x,y,z) = \sqrt{xyz}$ in the direction of $\vec{v} = \langle -1, -2, 2 \rangle$ at the point $(3, 2, 6)$

$$\frac{\partial f}{\partial v} = \frac{\vec{v} \circ \nabla f(x,y,z)}{|\vec{v}|}$$

$$\nabla f = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \\ \partial f / \partial z \end{bmatrix} = \begin{bmatrix} \sqrt{yz} / 2\sqrt{x} \\ \sqrt{xz} / 2\sqrt{y} \\ \sqrt{xy} / 2\sqrt{z} \end{bmatrix}$$

$$\frac{\partial f}{\partial x}\left(\sqrt{xyz}\right) = (xyz)^{1/2} = \sqrt{yz} \cdot \frac{1}{2} x^{-1/2}$$

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$$f(x,y,z) = \sqrt{xyz}$$

$$\frac{\partial}{\partial x}\left(\sqrt{xyz}\right) = \frac{1}{2}\,\frac{1}{\sqrt{x}}\,\sqrt{yz}$$

$$= \frac{\sqrt{yz}}{2\sqrt{x}}$$

$$\frac{\partial}{\partial y}\left(\sqrt{xyz}\right) = \frac{\sqrt{xz}}{2\sqrt{y}}$$

$$\frac{\partial}{\partial z}\left(\sqrt{xyz}\right) = \frac{\sqrt{xy}}{2\sqrt{z}}$$

$$\nabla f = \begin{bmatrix} \sqrt{12}/2\sqrt{3} \\ \sqrt{18}/2\sqrt{2} \\ \sqrt{\cancel{6}}/2\sqrt{\cancel{6}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3/2 \\ 1/2 \end{bmatrix} \circ \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} = -1 + -3 + 1$$

$$= -3 \rightarrow \frac{-3}{3} = \boxed{-1}$$

$$\left\| \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} \right\| = \sqrt{(-1)^2 + (-2)^2 + (2)^2}$$

$$= \sqrt{1 + 4 + 4}$$

$$= 3$$

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