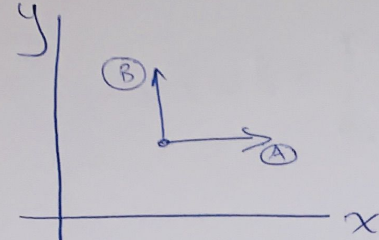
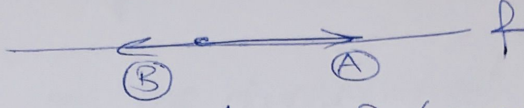
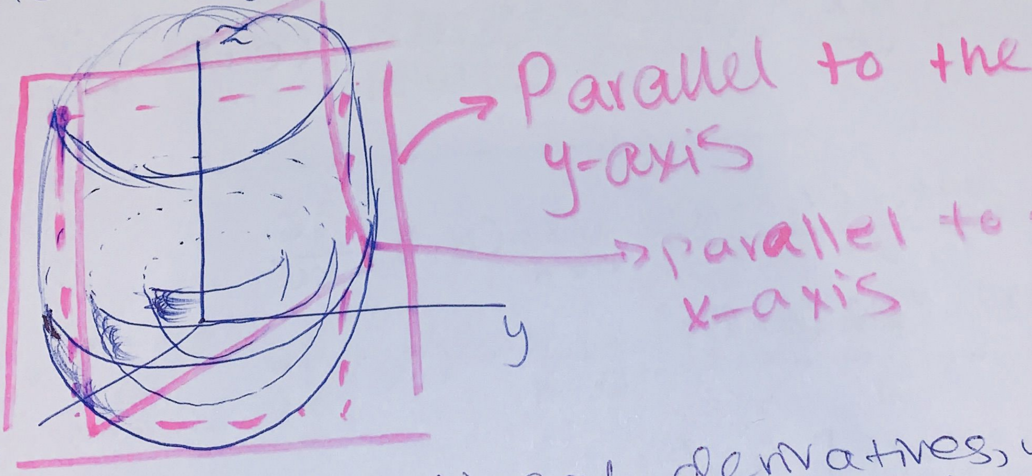
Directional Derivatives

Before, we could only ask...

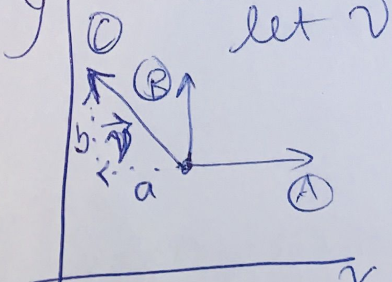




how does a step in ~~the~~ A or B (parallel to axes) affect the function? Analagous to "slicing"/tracing $f(x,y,..)$ parallel to the axes



but with directional derivatives, we ask: how does a step in C affect $f(x,y)$?



let $\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, so $\underbrace{h}_{.00001}\vec{v} = \begin{bmatrix} -h \\ 2h \end{bmatrix}$

$$\nabla_{\vec{v}} f(x,y) = -\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}$$

if $\vec{w} = \begin{bmatrix} a \\ b \end{bmatrix}$, $\nabla_{\vec{w}} f = a\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$

DIRECTIONAL DERIVATIVES

$$\nabla_{\vec{w}} f = a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y}, \text{ where } \vec{w} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\therefore, \nabla_{\vec{w}} f = \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\vec{w}} \cdot \underbrace{\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}}_{\nabla f} = \vec{w} \cdot \nabla f$$

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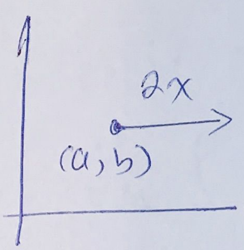
formal definition on the directional derivative

$$\frac{\partial f}{\partial x}(a,b) = \cancel{\lim_{h \to 0} [illegible]} = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$



$$\frac{\partial f}{\partial x}(a,b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$

$$\frac{\partial f}{\partial x}(\vec{a}) = \lim_{h \to 0} \frac{f(\vec{a} + h\hat{i}) - f(\vec{a})}{h}$$

$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ direction we're moving in

$$\frac{\partial f}{\partial x}(\vec{a}) = \lim_{h \to 0} \frac{f(\vec{a} + h\vec{v}) - f(\vec{a})}{h}$$

$$\nabla_{\vec{v}} f(x,y) = \frac{\nabla f \cdot \vec{v}}{\|\vec{v}\|}$$

Directional Derivatives

Directional derivative tells us:

$$\nabla_{\vec{v}} f(a,b) = \nabla f(a,b) \circ \vec{v}$$

$$= \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \circ \begin{bmatrix} a \\ b \end{bmatrix}$$

<u>Why</u> is the gradient the direction of steepest ascent? Well,...

$\max\limits_{\|\vec{v}\|=1} \left(\nabla f(a,b) \circ \vec{v} \right)$ is what we're looking for $\to \dfrac{\nabla f(a,b)}{\|\nabla f(a,b)\|}$

$\therefore$, if $\vec{v} = \vec{\nabla} f(a,b)$, then

$$\nabla_{\vec{v}} f = \nabla f \circ \vec{v} = \frac{\nabla f \circ \nabla f}{\|\nabla f\|} = \frac{\|\nabla f\|^2}{\|\nabla f\|}$$

$$= \|\nabla f\|$$