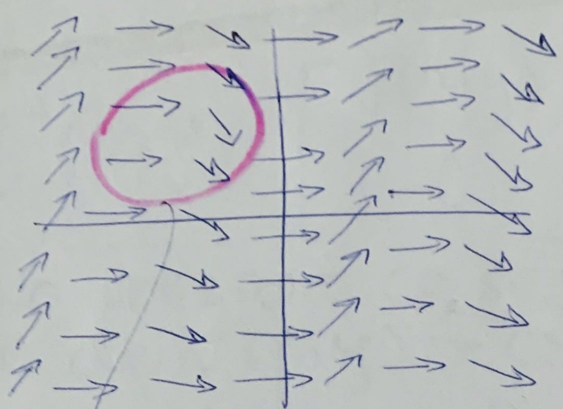
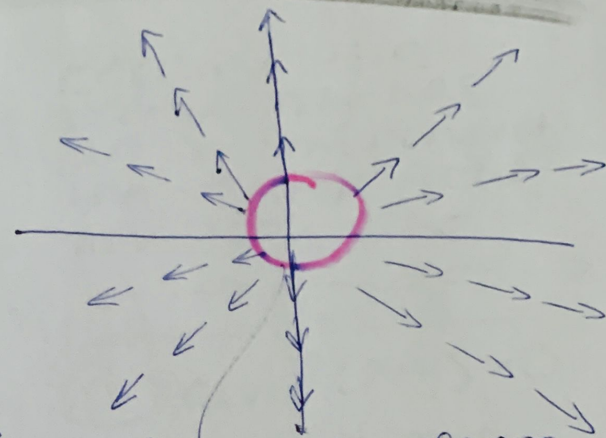
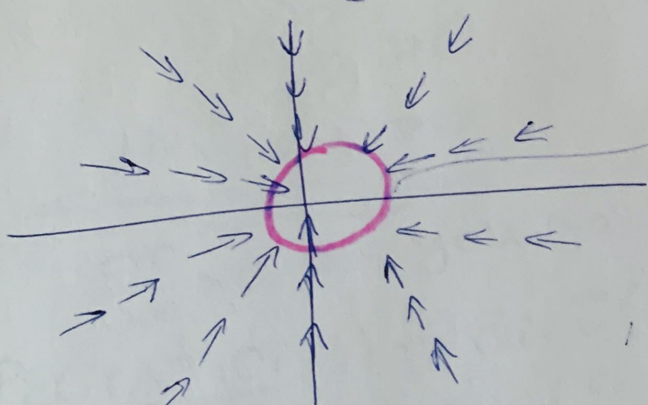
DIVERGENCE



→ density remains constant
→ molecules (same # go in as go out)
→ divergence is 0

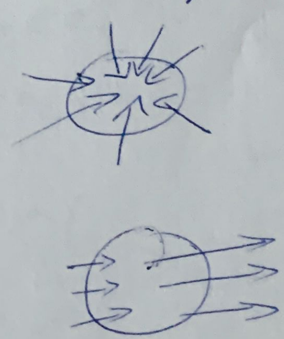


→ density decreases
→ molecules diverge
→ divergence is positive

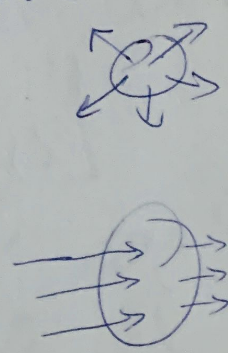


→ density increases
→ molecules all converge at the origin
→ divergence is negative

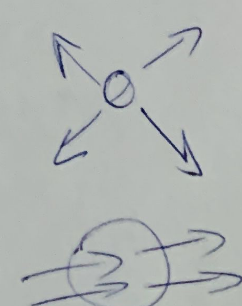
$div(\vec{v}(x,y)) > 0$



$div(\vec{v}(x,y)) > 0$



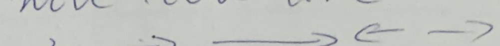
$div(\vec{v}(x,y)) = 0$



DIVERGENCE

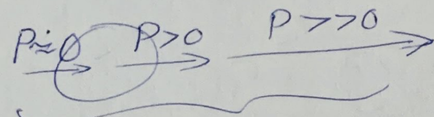
$$\vec{v}(x,y) = \begin{bmatrix} P(x,y) \\ 0 \end{bmatrix}$$

Because the vector only has an x-component, the vector field will look like:



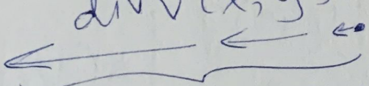
Two possible cases for $\operatorname{div} \vec{v}(x,y) > 0$

C1 $\operatorname{div} \vec{v}(x,y) > 0$



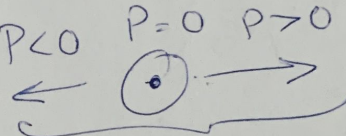
$$\frac{\partial P}{\partial x} > 0$$

$\operatorname{div} \vec{v}(x,y) > 0$



$\frac{\partial P}{\partial x} > 0$ as you go to the right

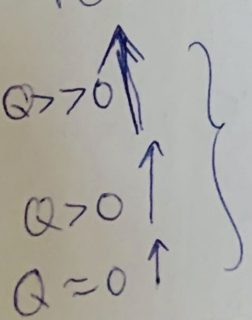
C2 $\operatorname{div} \vec{v}(x,y) > 0$



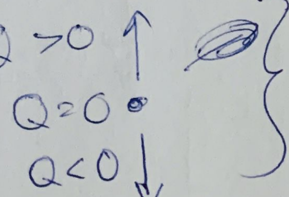
$$\frac{\partial P}{\partial x} > 0$$

EXPECTATION: If $\operatorname{div} \vec{v}(x,y) > 0$ for $\vec{v}(x,y) = \begin{bmatrix} P(x,y) \\ 0 \end{bmatrix}$ then $\frac{\partial P}{\partial x} > 0$ & if $\vec{v}(x,y) = \begin{bmatrix} 0 \\ Q(x,y) \end{bmatrix}$, $\frac{\partial Q}{\partial y} > 0$ for $\operatorname{div} \vec{v}(x,y) > 0$

Two possible cases for $\vec{v}(x,y) = \begin{bmatrix} 0 \\ Q(x,y) \end{bmatrix}$



$$\frac{\partial Q}{\partial y} > 0$$



$$\frac{\partial Q}{\partial y} > 0$$

DIVERGENCE

$$\text{div}\,\vec{v}(x,y) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \quad \text{for } \vec{v}(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\vec{v}(x,y) = \begin{bmatrix} xy \\ y^2 - x^2 \end{bmatrix} ; \quad \text{div}\,\vec{v}(x,y) = \left\{ \begin{matrix} \frac{\partial}{\partial x}(xy) \\ + \\ \frac{\partial}{\partial y}(y^2 - x^2) \end{matrix} \right\}$$

$$\text{div}\,\vec{v}(x,y) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = \left\{ \begin{matrix} y \\ + \\ 2y \end{matrix} \right\} = 3y$$

$$\vec{v}(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

can be interpreted as "fluid flow only affected by y value"

$$\nabla \circ \vec{v} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \circ \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

$$\vec{v}(x,y,z) = \begin{bmatrix} P(x,y,z) \\ Q(x,y,z) \\ R(x,y,z) \end{bmatrix} ; \quad \nabla \circ \vec{v} = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \circ \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$