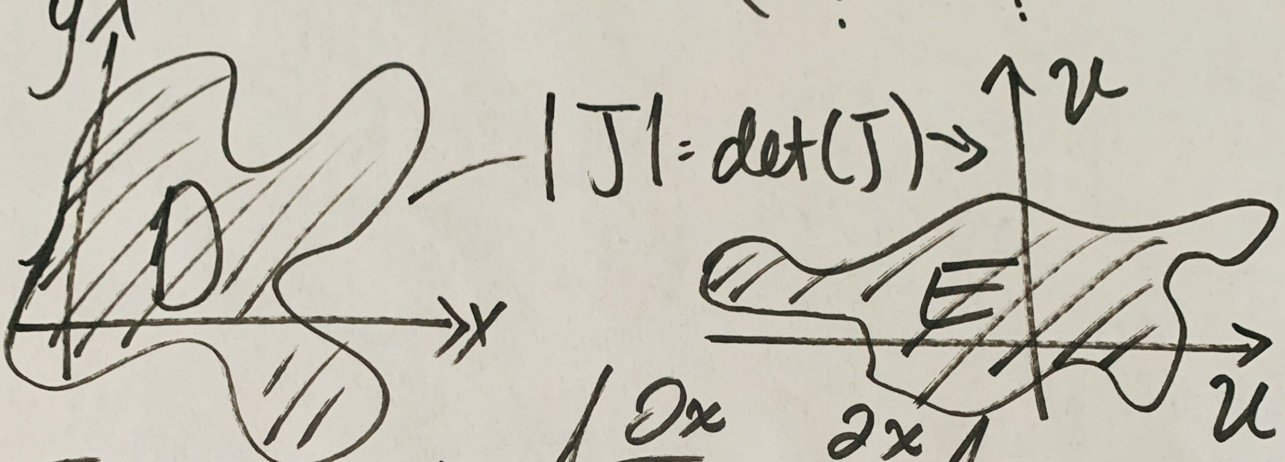
JACOBIAN

Jacobian Matrix: $J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\ \vdots & \vdots & \end{pmatrix}$



$$|J| = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \Rightarrow \det(J) =$$

$$\frac{\partial x}{\partial u}\cdot\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\cdot\frac{\partial y}{\partial u} = |J| = \det(J)$$

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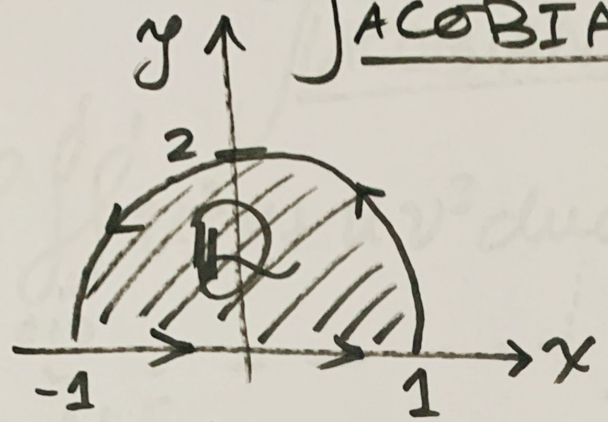
let: $\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\}$ POLAR

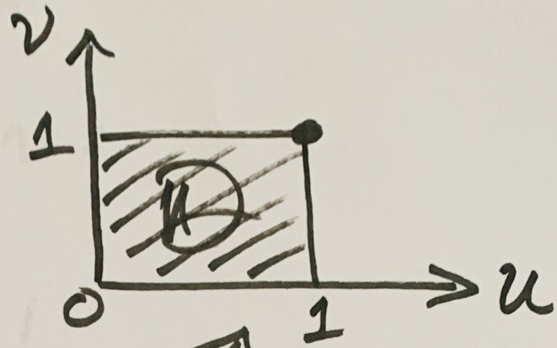
$$|J| = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} \Rightarrow \det(J)$$

$$= r\cos^2\theta + r\sin^2\theta = r(\cos^2\theta + \sin^2\theta)$$

$= r$ SCALE FACTOR

JACOBIAN II





$$\boxed{\begin{aligned} x &= u^2 - v^2 \\ y &= 2uv \end{aligned}}$$

limits of integration:

$0 \le u \le 1$

$0 \le v \le 1$

$$\iint_D y\, dA = \iint_{D'} 2uv\, |J|\, dA$$

$$|J| = \frac{\partial(x,y)}{\partial(u,v)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} 2u & -2v \\ 2v & 2u \end{bmatrix}$$

$$\det(J) = 4u^2 + 4v^2 = 4(u^2+v^2)$$

$$dA = du\,dv$$

$$\int_0^1\int_0^1 2uv(4(u^2+v^2))\, du\, dv =$$

D

$$8\int_0^1\int_0^1 uv(u^2+v^2)\, du\, dv =$$

$$8\int_0^1\int_0^1 u^3v + uv^3\, du\, dv$$

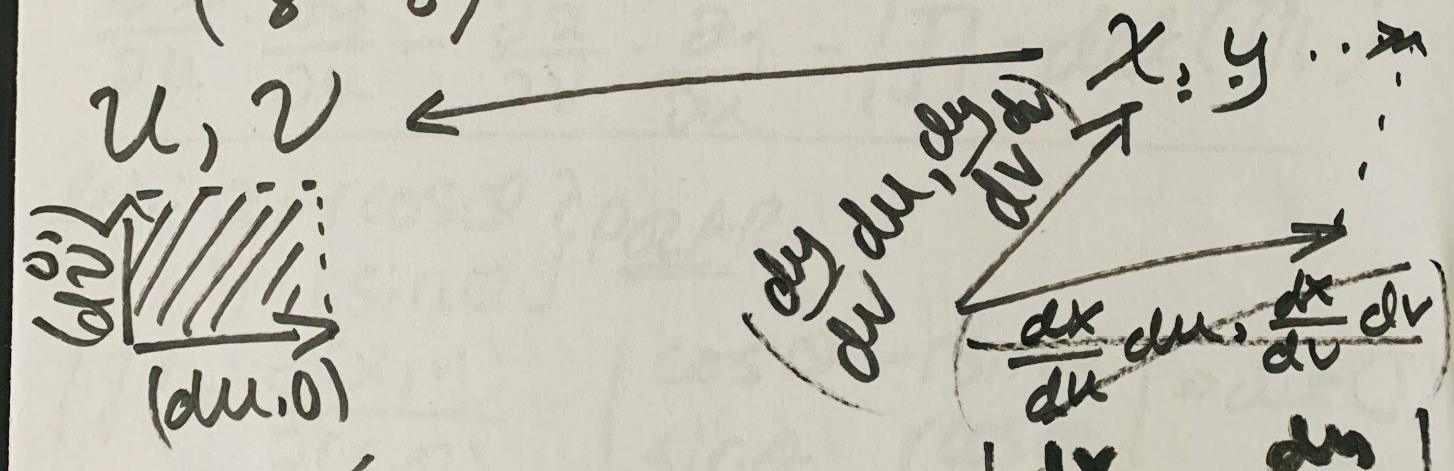
JACOBIAN III

$$8\int_0^1\int_0^1 u^3 v + u v^3 \, du\, dv =$$

$$8\int_0^1 \left[\frac{u^4}{4} v + \frac{u^2}{2} v^3 \right]_0^1 dv =$$

$$8\int_0^1 \left(\frac{1}{4} v + \frac{1}{2} v^3 \right) dv = 8\left(\frac{v^2}{2} \cdot \frac{1}{4} + \frac{v^4}{4} \cdot \frac{1}{2} \right)\Big|_0^1$$

$$= 8\left(\frac{1}{8} + \frac{1}{8} \right) = 2$$



Area: $du\,dv$

$$\text{Area} = \begin{vmatrix} \frac{dx}{du} & \frac{dy}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} \end{vmatrix}$$