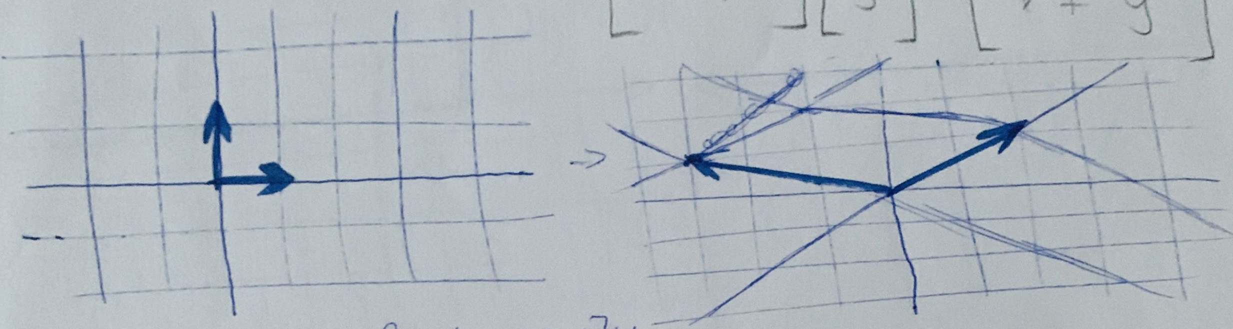
JACOBIAN

Jacobian Matrix: $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x-3y \\ x+y \end{bmatrix}$



Properties of linearity

A vector-valued function L is linear if:

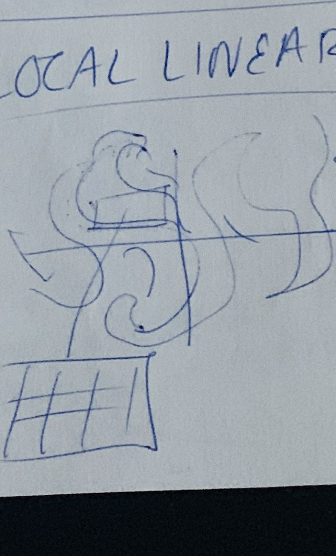
$L(\underbrace{a\vec{v}}_{\text{scaling a vector}}) = a\underbrace{L(\vec{v})}_{\text{transforming a vector}}$ } transforming a scaled vector = scaling a transformed vector

$L(\vec{v}+\vec{w}) = L(\vec{v}) + L(\vec{w})$ } transforming sum of vectors = summing transformed vectors

$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = L\left(x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$ } combines the above two into a linear combination!!

$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+\sin(y) \\ y+\sin(x) \end{bmatrix}$ ★ LOCAL LINEARITY!!

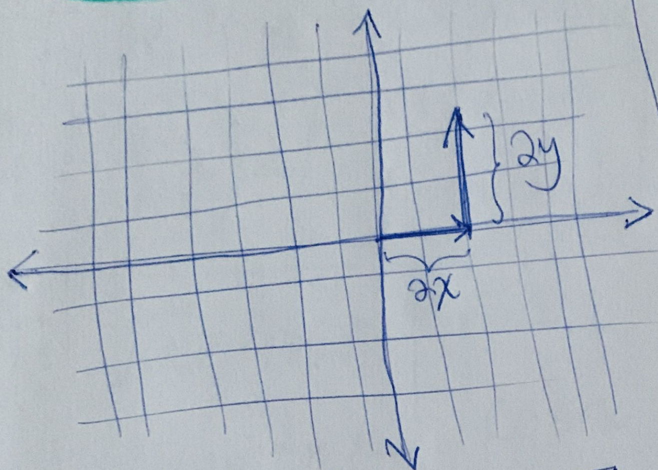
$\begin{bmatrix} \frac{\pi}{2} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\pi}{2} \\ \sin(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} \pi/2 \\ 1 \end{bmatrix}$

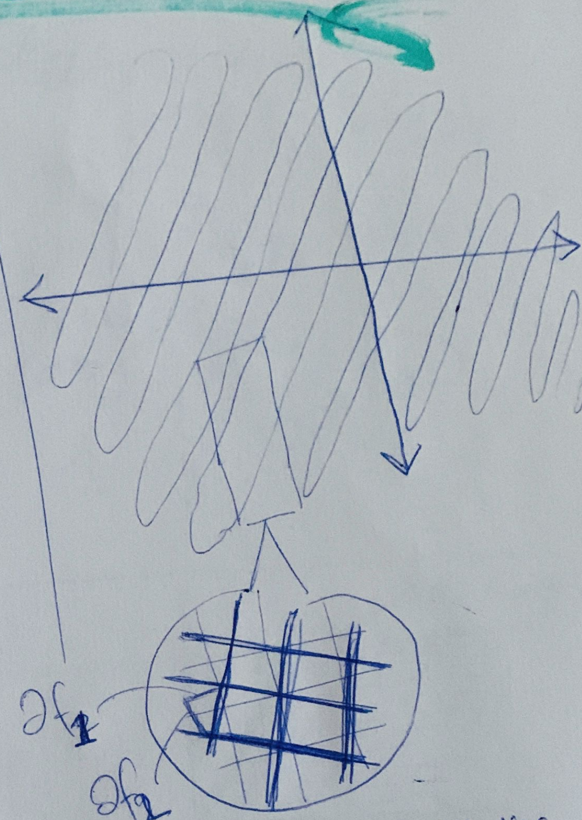


← NOT LINEAR TRANSFORMATION, but zoom in closely...

1

JACOBIAN





$$\begin{bmatrix} \partial f_1/\partial x & \partial f_2/\partial x \\ \partial f_1/\partial y & \partial f_2/\partial y \end{bmatrix}$$

7 → **MISTAKE**: Remember, the first column is the first basis vector and the second column is the second basis vector

$$\begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix}$$ ← JACOBIAN MATRIX

$f(x, y)$ } plug in a specific point on $f(x,y)$ to find what the zoomed-in transformation looks like

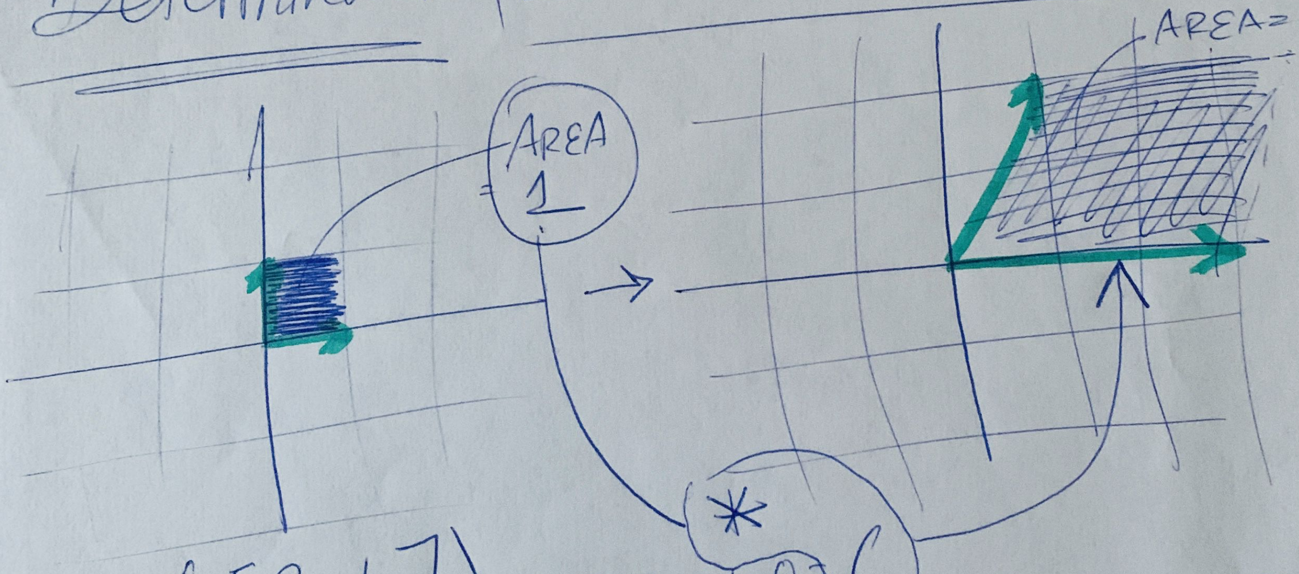
JACOBIAN

Find the Jacobian Matrix for

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+\sin(y) \\ y+\sin(x) \end{bmatrix}; \quad J = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix}$$

$$\begin{bmatrix} 1 & \cos y \\ \cos x & 1 \end{bmatrix} = J_f$$

The Jacobian Determinant



$$\det\left(\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\right) = 3 \cdot 2 - 1 \cdot 0 = 6$$

① The Jacobian describes non-linear transformations at a LOCAL scale:

$$\begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 1 & \cos y \\ \cos x & 1 \end{bmatrix}\right) = 1 - \cos(x)\cos(y)$$