

~~DO~~ Vectors & Linear Equations

III

SUMMARY: We now introduce the Matrix Picture & the associated notation.

DO NOW: Write the coefficient matrix of this system:

$$\begin{cases} x + 2y + 3z = 6 \\ 2x + 5y + 2z = 4 \\ 6x - 3y + z = 2 \end{cases} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & -3 & 1 \end{bmatrix}$$

LESSON: Understand the (1) Row Picture, (2) Column Picture of this system

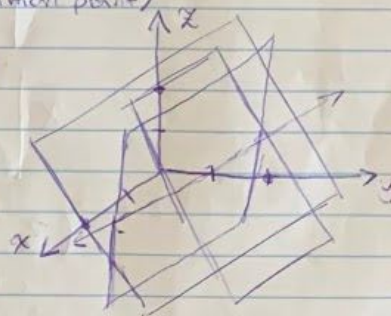
$$\begin{cases} -3x + 2y - 5z = -14 \\ 2x - 3y + 4z = 10 \\ x + y + z = 4 \end{cases}$$

① ROW PICTURE (3 planes intersect at a common point)

$$-3x + 2y - 5z = -14 \leftarrow \begin{matrix} (0, -7, 0) \\ (14/3, 0, 0) \\ (0, 0, 14/5) \end{matrix}$$

$$2x - 3y + 4z = 10 \leftarrow \begin{matrix} (5, 0, 0) \\ (0, 10/3, 0) \\ (0, 0, 10/4) \end{matrix}$$

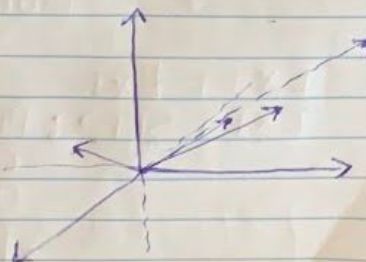
$$x + y + z = 4 \leftarrow \begin{matrix} (4, 0, 0) \\ (0, 4, 0) \\ (0, 0, 4) \end{matrix}$$



② COLUMN PICTURE

$$\begin{cases} -3x + 2y - 5z = -14 \\ 2x - 3y + 4z = 10 \\ 1x + 1y + 1z = 4 \end{cases}$$

$$x \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + z \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -14 \\ 10 \\ 4 \end{bmatrix}$$



③ MATRIX PICTURE

$$\begin{aligned} -3x + 2y - 5z &= -14 \\ 2x - 3y + 4z &= 10 \\ 1x + 1y + 1z &= 4 \end{aligned} \rightarrow \begin{bmatrix} -3 & 2 & -5 \\ 2 & -3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

coefficient matrix

$$Ax = b$$

↑ We want this

$$\underbrace{\begin{bmatrix} -3 & 2 & -5 \\ 2 & -3 & 4 \\ 1 & 1 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_x = \underbrace{\begin{bmatrix} -14 \\ 10 \\ 4 \end{bmatrix}}_b$$

*The Fundamental Question of Linear Algebra: How to solve $Ax = b$?

$$x \begin{bmatrix} -3 & 2 & -5 \\ 2 & -3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$x \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + z \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -14 \\ 10 \\ 4 \end{bmatrix}$$

$(3, 0, 1)$ turns to be the solution. we verify by plug and checking in.

~~EXIT EX~~

EXIT SLIP: Plug in $(3, 0, 1)$ for x and check if your calculations are right.

$$3 \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -14 \\ 10 \\ 4 \end{bmatrix} \checkmark$$