LINEAR ALGEBRA #13

Vectors & Linear Equations XIII

Do Now: Span$(\vec{u},\vec{v}) = \mathbb{R}^2$

LESSON: But that's the normal case. Did you know there's more? We examine each scenario:

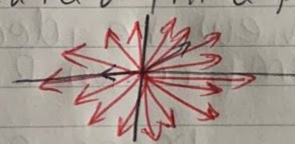
Scenario #1: Span$(\vec{u},\vec{v}) = \mathbb{R}^2$

Notation: Span$(\vec{u},\vec{v}) = \mathbb{R}^2$

Algebra: $c\vec{u}+d\vec{v}$ ← plug in different values of c,d & u,v like $(\vec{u},\vec{v}) = \left(\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix}\right)$ & $(c,d) = \{(1,1)(0,1),(2,1),(5,2)\}$

Translation: All linear combinations of $c\vec{u}+d\vec{v}$ fill a plane.

Geometric:



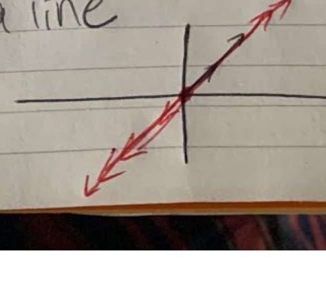
Scenario #2:

Notation: Span$(\vec{u},\vec{v}) = \mathbb{R}^1$

Algebra: Let $\vec{v} = c\vec{u}$, so that $\vec{v}$ is a multiple of $\vec{u}$. For example, $(\vec{u},\vec{v}) = \left(\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}2\\4\end{bmatrix}\right)$ Now we have that all linear combinations are represented by $cu+dv \Rightarrow c\begin{bmatrix}1\\2\end{bmatrix}+d\begin{bmatrix}2\\4\end{bmatrix}$. Try $(c,d) = \{(0,1),(1,0),(1,1),(2,1),(2,0)\}$ You end up in a line. No escape!

Translation: All linear combinations of $c\vec{u}+d\vec{v}$ fill a line

Geometric

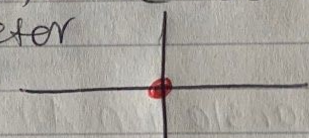


Scenario #3:

Notation: Span$(\vec{u},\vec{v}) = \mathbb{R}^0$

Algebra: Let's say $\vec{u} = \vec{v} = \begin{bmatrix}0\\0\end{bmatrix}$! So, now if we have $cu + dv$, I have $c\begin{bmatrix}0\\0\end{bmatrix} + d\begin{bmatrix}0\\0\end{bmatrix}$. But no matter what (c,d) I plug in, I get 0!

Translation: All linear combinations of $c\vec{u} + d\vec{v}$ fill a origin if $\vec{u},\vec{v}$ are both the zero vector

Geometric: 

EXIT SLIP: Review the idea of a linear combination. Remember,

linear combination

scalar multiplication $\oplus$ vector addition

$$c\vec{u} + d\vec{v}$$

$$\Updownarrow$$

$$\text{Span}(\vec{u},\vec{v}) = \mathbb{R}^k$$

$$\Updownarrow$$