LINEAR ALGEBRA #14

Vectors & Linear Equations XIV

Do Now: $\text{Span}(u, v, w) = \mathbb{R}^3$ or $\mathbb{R}^2$ or $\mathbb{R}^1$ or $\mathbb{R}^0$

Lesson: Case I:

NOTATION: $\text{span}(u, v, w) = \mathbb{R}^3$

TRANSLATION: All linear combinations of $cu+dv+ew$ fill all of 3D space

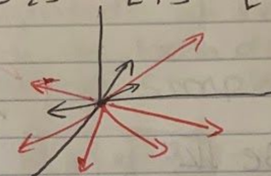
ALGEBRA: $cu+dv+ew$

$(u,v,w) = \left(\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right)$

$(a,b,c) = (-1, 0, 1)$

$-1 \cdot \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + 0 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

GEOMETRY:



Case II:

NOTATION: $\text{span}(u, v, k) = \mathbb{R}^2$

TRANSLATION: All linear combinations of $cu+dv+ew$ fill the 2D plane

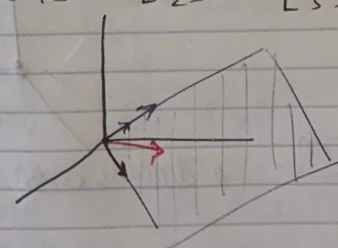
ALGEBRA: $cu+dv+ew$

$(u,v,w) = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}\right)$

$(c,d,e) = (+1, 0, 0)$

$1 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 0 \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + 0 \cdot \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}$

GEOMETRY:



Case III:

NOTATION: $\text{Span}(u, v, k) = \mathbb{R}^1$

TRANSLATION: All linear combinations of $cu+dv+ek$ fill the line

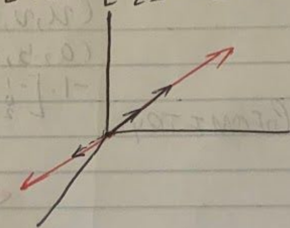
ALGEBRA: $cu+dv+ek$

$$(u, v, k) = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right)$$

$$(c, d, e) = (-1, 1, 1)$$

$$-1 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + 1 \cdot \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

GEOMETRY:



Case IV:

NOTATION: $\text{Span}(u, v, k) = \mathbb{R}^0$

TRANSLATION: All linear combinations of $cu+dv+ek$ fill a point

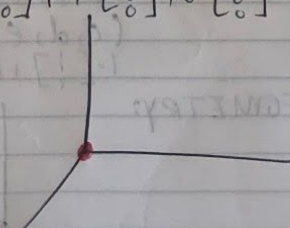
ALGEBRA: $cu+dv+ek$

$$(u, v, k) = \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$(c, d, e) = (2, 4, 6)$$

$$2 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 4 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 6 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

GEOMETRY:



Exit Slip: $\mathbb{R}^n$ = n-dimensions