LINEAR ALGEBRA #15

Vectors & Linear Equations XV

Do Now: Why can't $\text{span}(u) = \mathbb{R}^2$? B/C it doesn't 'punch' out of the line. Likewise, $\text{span}(u,v) \neq \mathbb{R}^{n>2}$ and $\text{span}(u,v,w) \neq \mathbb{R}^{n>3}$

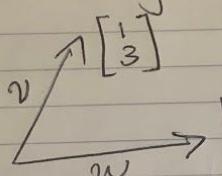
Lesson: Introduction to the dot product. There are two main definitions:

$$v \cdot w = \sum_{i}^{n} v_i w_i$$

$$v \cdot w = |v||w|\cos\theta$$

$|v|$ → magnitude of vector v

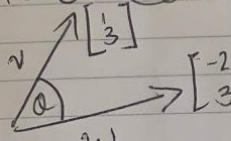
Today, we look at both definitions. We begin with the simpler definition:



$$\begin{bmatrix}1\\3\end{bmatrix} \cdot \begin{bmatrix}-2\\3\end{bmatrix} = (1)(-2) + (3)(3) = 7$$

But what does this 7 mean? What does it tell us?

To find out, I extend my question: What is the angle between the two vectors?



$$7 = |v||w|\cos\theta$$

$$\frac{7}{|v||w|} = \cos\theta$$

$$\frac{7}{(\sqrt{1^2+3^2})(\sqrt{(-2)^2+(3)^2})} = \left(\frac{7}{\sqrt{10}\cdot\sqrt{13}}\right)\cos^{-1} = \theta$$

Exit Slip: What are two things we can do with the dot product?

- Find magnitude — Find angles