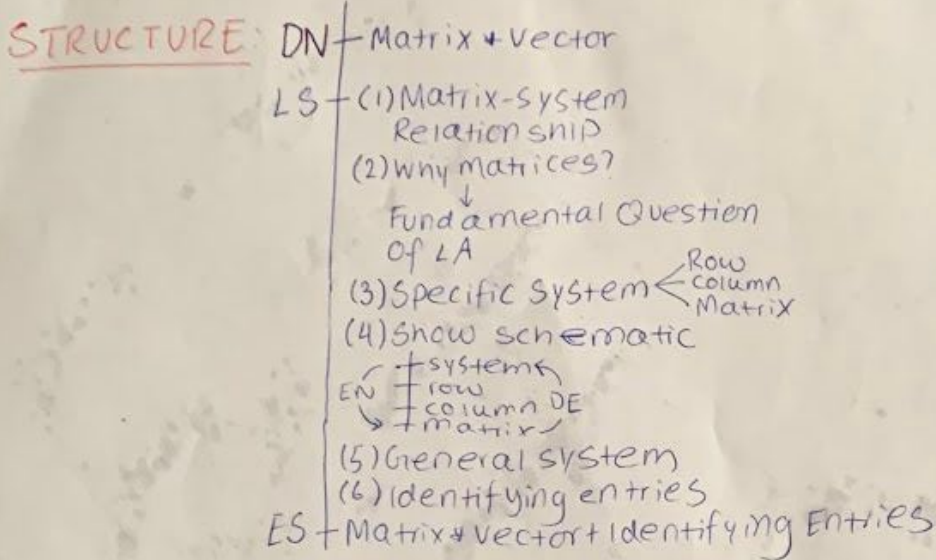
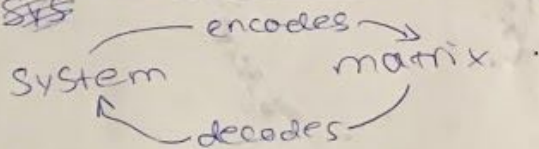


# VECTORS & LINEAR EQUATIONS - IV



Do Now: 
$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 19 \\ 29 \end{bmatrix}$$

LESSON: 1. Matrix-system relation



2. Why matrices?

- compact
- designed to answer the FUNDAMENTAL QUESTION OF LA:  
How to solve system of linear equations?  
How to solve  $AX=b$ ? ← same question!

### 3. Specific System

$$\left. \begin{array}{l} x+y+z=2 \\ x+2y+z=3 \\ 2x+3y+2z=5 \end{array} \right\} \text{system}$$

GOAL: Solve system of Linear Equations for  $x, y, z$

STRUCTURE:  $\left\{ \begin{array}{l} \text{system} \\ \text{row picture} \\ \text{column picture} \\ \text{matrix picture} \end{array} \right.$

ROW PICTURE:

$$\begin{array}{l} x+y+z=2 \\ x+2y+z=3 \\ 2x+3y+2z=5 \end{array}$$

COLUMN PICTURE:

$$x \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + z \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

MATRIX PICTURE:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

ENCODES INTO

DECODES INTO

### 4. Schematic

We thus understand why  
"Solve the system of Linear Equations for  $x, y, z$ "  
is the same as  
"Solve  $Ax=b$  for  $x$ "

ENCODE  $\left\{ \begin{array}{l} \text{system} \\ \text{row} \\ \text{column} \\ \text{matrix} \end{array} \right.$  DECODE

### 5. General System

$$\left. \begin{array}{l} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{array} \right\} \text{system}$$

ROW PICTURE:

$$\begin{array}{l} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{array}$$

## COLUMN PICTURE:

$$x \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

## MATRIX PICTURE:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

## B. Identifying Entries

$$\left. \begin{array}{l} \begin{bmatrix} a \\ A_1 & A_2 & A_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ x \begin{bmatrix} A_1 \end{bmatrix} + y \begin{bmatrix} A_2 \end{bmatrix} + z \begin{bmatrix} A_3 \end{bmatrix} \end{array} \right\} \begin{array}{l} \text{Matrix} \\ \text{vector} \\ \text{multiplication} \\ \text{general} \end{array}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

EXIT SLIP: Identify  $(Ax)_{21}$

$$\begin{array}{ccc} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} & = & \begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix} \\ A & x & Ax & \end{array} \quad \text{--- } (Ax)_{21}$$