VECTORS & LINEAR EQUATIONS — IV

STRUCTURE:

- DN — Matrix + Vector
- LS — (1) Matrix-System Relationship
  - (2) Why matrices? → Fundamental Question of LA
  - (3) Specific System < Row / Column / Matrix
  - (4) Show schematic
    - EN → system, row, column, matrix ← DE
  - (5) General system
  - (6) Identifying entries
- ES — Matrix + Vector + Identifying Entries

DO NOW:

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 19 \\ 29 \end{bmatrix}$$

LESSON:

1. Matrix-system relation

~~SFS~~

$$\begin{tikzcd} \text{System} \arrow[r, bend left, "\text{encodes}"] & \text{matrix} \arrow[l, bend left, "\text{decodes}"] \end{tikzcd}$$

2. Why matrices?

- compact
- designed to answer the FUNDAMENTAL QUESTION OF LA:

How to solve system of linear equations?

How to solve $Ax=b$? ← same question!

3. Specific System

$$\left.\begin{aligned} x+y+z&=2\\ x+2y+z&=3\\ 2x+3y+2z&=5 \end{aligned}\right\}\text{System}$$

<u>GOAL</u>: Solve system of Linear Equations for x, y, z

<u>STRUCTURE</u>:
- system
- row picture
- column picture
- matrix picture

ROW PICTURE:

$$\begin{aligned} x+y+z&=2\\ x+2y+z&=3\\ 2x+3y+2z&=5 \end{aligned}$$

ENCODES INTO

DECODES INTO

COLUMN PICTURE:

$$x\begin{bmatrix}1\\1\\2\end{bmatrix}+y\begin{bmatrix}1\\2\\3\end{bmatrix}+z\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$$

MATRIX PICTURE:

$$\begin{bmatrix}1&1&1\\1&2&1\\2&3&2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$$

4. Schematic

We thus understand why
"Solve the system of Linear Equations for x, y, z"
is the same as
"Solve $Ax=b$ for x"

- system
- row
- column
- matrix

ENCODE DECODE

5. General System

$$\left.\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}\right\}\text{System}$$

ROW PICTURE:

$$\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}$$

COLUMN PICTURE:

$$x\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

MATRIX PICTURE:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

6. Identifyng Entries

$$\begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x\begin{bmatrix} A_1 \end{bmatrix} + y\begin{bmatrix} A_2 \end{bmatrix} + z\begin{bmatrix} A_3 \end{bmatrix}$$

Matrix vector multiplication general

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

EXIT SLIP: Identify $(Ax)_{21}$

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix}}_{Ax} \text{— } (Ax)_{21}$$