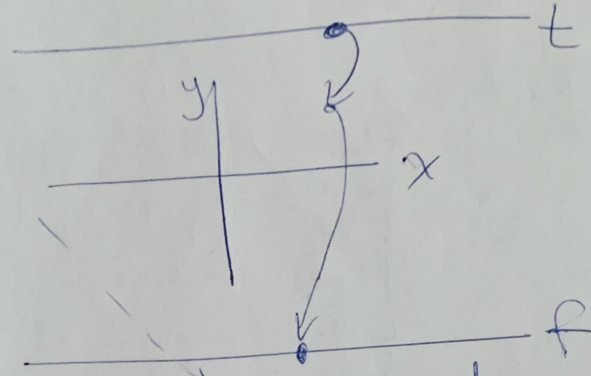
MULTIVARIABLE CHAIN RULE

$f(x,y) = x^2 y$

$x(t) = \cos(t)$

$y(t) = \sin(t)$

$f(x(t), y(t))$



$\frac{d}{dt} f((x(t), y(t))$

$\frac{d}{dt} f((\cos(t), \sin(t))$

$\frac{d}{dt} (\cos t)^2 \sin(t)$

$\frac{d}{dt} (\cos^2 t \sin t)$

$\frac{d}{dt}(\cos^2 t)\sin t + \frac{d}{dt}(\sin t)\cos^2 t$

$2\cos t(-\sin^2 t) + \cos^3 t$

$-2\sin^2 t\cos t + \cos^3 t$

$\frac{\partial f}{\partial x} = 2xy$ | $\frac{dx}{dt} = -\sin(t)$

$\frac{\partial f}{\partial y} = x^2$ | $\frac{dy}{dt} = \cos(t)$

$\underbrace{\cos^2(t)}\cos(t) + 2\cos(t)\sin(t)(-\sin(t))$

$\left(\frac{\partial f}{\partial y}\right)(\cos(t)) + \left(\frac{\partial f}{\partial x}\right)(-\sin(t))$

$\left(\frac{\partial f}{\partial y}\right)\left(\frac{dy}{dt}\right) + \left(\frac{\partial f}{\partial x}\right)\left(\frac{dx}{dt}\right)$

$\left[\begin{matrix}\cancel{\frac{\partial f}{\partial y}} \\ \cancel{\frac{\partial f}{\partial}}\end{matrix}\right.$

MULTIVARIABLE CHAIN RULE

$$\left(\frac{\partial f}{\partial y}\right)\left(\frac{dy}{dt}\right) + \left(\frac{\partial f}{\partial x}\right)\left(\frac{dx}{dt}\right)$$

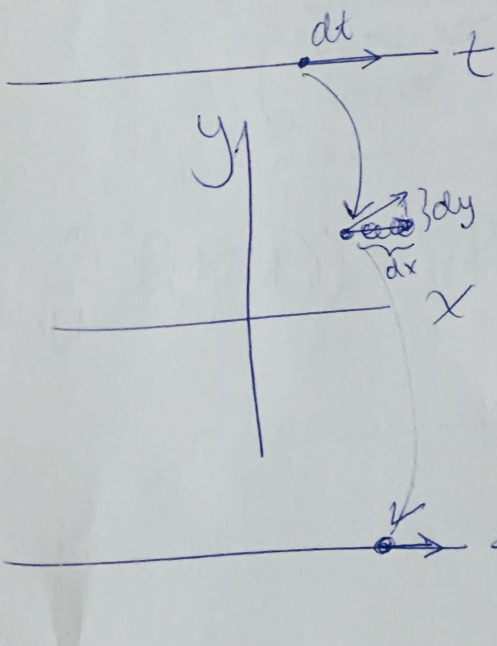
$$\begin{bmatrix}\frac{\partial f}{\partial x}\\ \frac{\partial f}{\partial y}\end{bmatrix} \cdot \begin{bmatrix}\frac{dx}{dt}\\ \frac{dy}{dt}\end{bmatrix}$$

let $f(x,y)$, then

$$\begin{bmatrix}\frac{\partial f}{\partial x}\\ \frac{\partial f}{\partial y}\end{bmatrix} \cdot \begin{bmatrix}\frac{dx}{dt}\\ \frac{dy}{dt}\end{bmatrix}$$

$\frac{\partial f}{\partial x}$??=

$$\boxed{\frac{d}{dt} f((x(t), y(t)) = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}}$$



$$dx = \underbrace{\frac{dx}{dt}}\, dt$$

how much does a tiny nudge in dt change ~~dx~~ x?

likewise, ...

$$dy = \frac{dy}{dt}\, dt$$

$$df = \frac{\partial f}{\partial x}\, dx$$

$$df = \frac{\partial f}{\partial y}\, dy$$

$\rightarrow$

$$df = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt}\, dt + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}\cdot dt$$

MULTIVARIABLE CHAIN RULE

Q: What does the multivariable chain rule look like in vector form?

$$\frac{d}{dt} f(x(t), y(t)) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\vec{v}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \; ; \; \cancel{\frac{d}{dt}\vec{v}(t) = \frac{\partial f}{\partial x} \cdot}$$

$$\frac{d\vec{v}}{dt} = \begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \end{bmatrix} \; ; \; \frac{d}{dt}\vec{v} = \underbrace{\begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix}}_{} \cdot \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$$

(MULTIVARIABLE CHAIN RULE, FOR VECTOR VALUED FUNCTION) $\Longleftrightarrow$ $\boxed{\nabla f \cdot \vec{v}'(t)}$

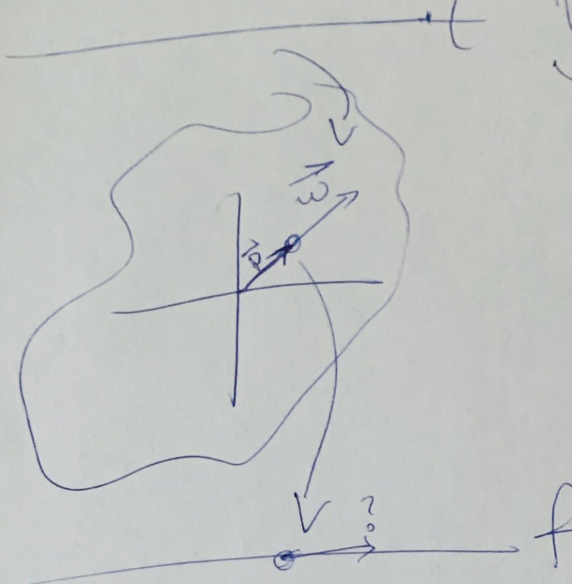
$$= \boxed{\cancel{\nabla f(\vec{v}'(t))}}$$

$$\frac{d}{dt} f(\vec{v}(t)) = \nabla f(\vec{v}(t)) \cdot \vec{v}'(t)$$

$$= \nabla f(\vec{v}(t)) \cdot \vec{v}'(t)$$

$$= \nabla f(\vec{v}(t)) \cdot \frac{d\vec{v}}{dt}$$

MULTIVARIABLE CHAIN RULE



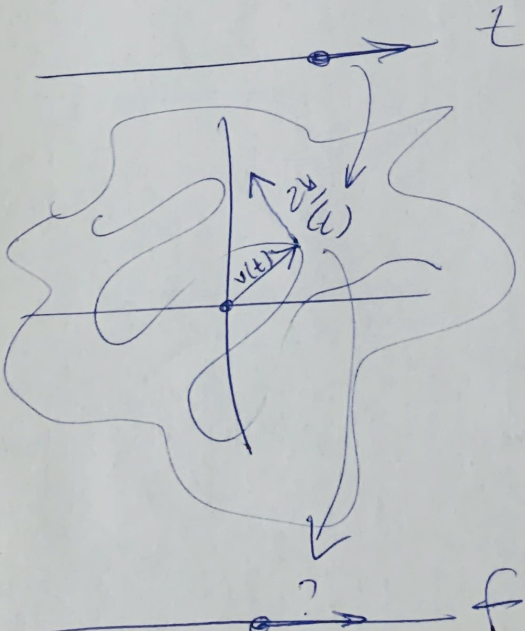
$$\underbrace{\nabla_{\vec{w}} f(\vec{p}) = \nabla f(\vec{p}) \cdot \vec{w}}$$

Directional Derivative

looks a lot like multivariable chain rule

$$\frac{d}{dt} f(\vec{v}(t)) = \nabla f(\vec{v}(t)) \cdot \vec{v}\,'(t) =$$

$$\nabla_{\vec{v}\,'(t)} f(\vec{v}(t))$$



MULTIVARIABLE CHAIN RULE & DIRECTIONAL DERIVATIVE