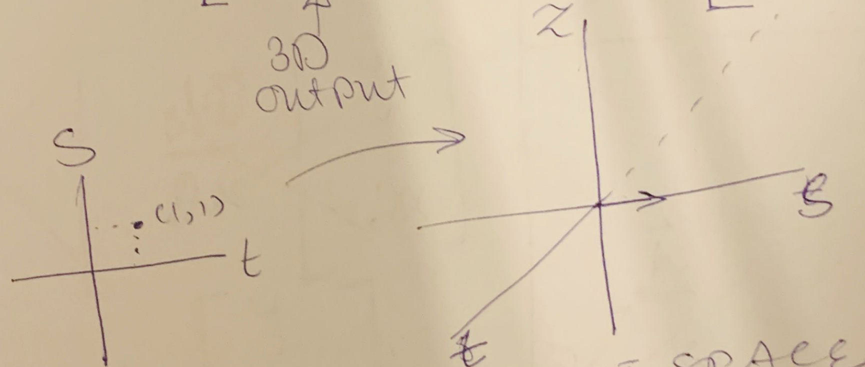
PARTIAL DERIVATIVES OF VECTOR-VALUED FUNCTIONS

$$\vec{v}(t,s) = \begin{bmatrix} t^2 - s^2 \\ st \\ ts^2 - st^2 \end{bmatrix} \Leftrightarrow \frac{\partial \vec{v}}{\partial t} = \begin{bmatrix} 2t \\ s \\ s^2 - 2ts \end{bmatrix}$$

2D input

3D output



INPUT SPACE

OUTPUT SPACE

$$\vec{v}(1,1) = \begin{bmatrix} 1 - \phi \\ (1)(1) \\ (1)(1) - (1)(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

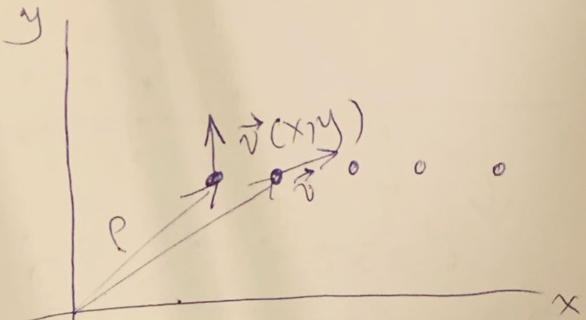
	PREDICT	REAL
$\frac{\partial \vec{v}}{\partial t} = \begin{bmatrix} 2t \\ s \\ s^2 - 2ts \end{bmatrix}$	$\begin{bmatrix} >0 \\ >0 \\ <0 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

movement in 3D space!!

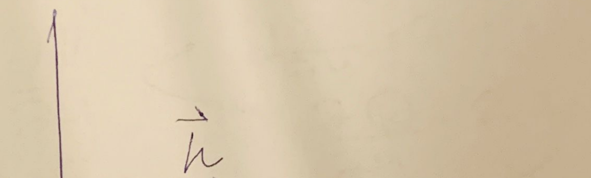
PARTIAL DERIVATIVE OF PARAMETRIC SURFACE

Partial derivative of vector fields:

$$\vec{v}(x,y) = \begin{bmatrix} xy \\ y^2 - x^2 \end{bmatrix}$$

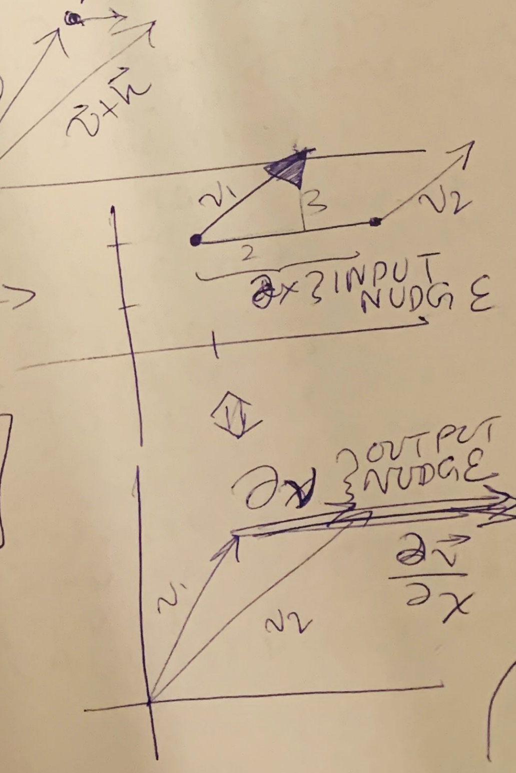


$$\cancel{\frac{\partial \vec{v}}{\partial t}} = \left[\ \cancel{\frac{d\vec{v}}{dx} = \begin{bmatrix} y \\ -2x \end{bmatrix}}\ \right]$$

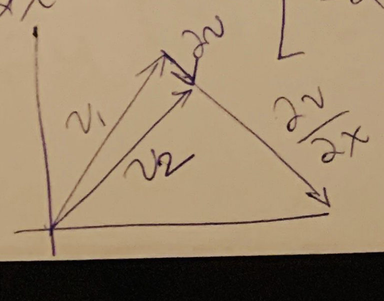


$$\frac{\partial \vec{v}}{\partial x}(x,y) = \begin{bmatrix} y \\ -2x \end{bmatrix}$$

$$\vec{v}(1,2) = \begin{bmatrix} (1)(2) \\ (2)^2 - (1)^2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Longleftrightarrow$$

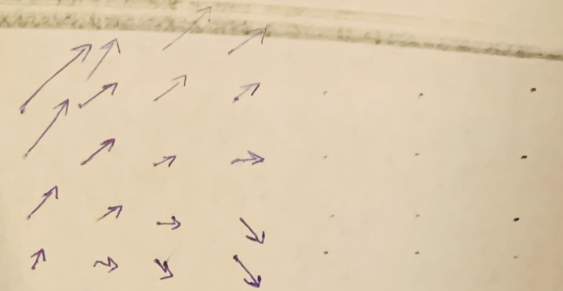


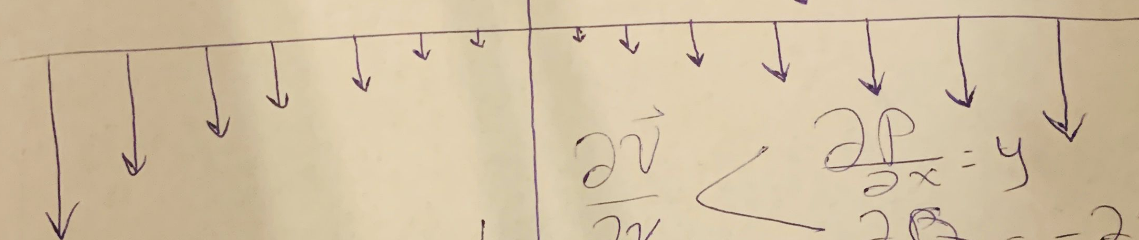
$$\frac{\partial v}{\partial x}(1,2) = \begin{bmatrix} 2 \\ -2(1) \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



PARTIAL DERIVATIVES OF VECTOR FIELDS

$$\vec{v}(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$





$$\vec{v}(x,y) = \begin{bmatrix} xy \\ y^2 - x^2 \end{bmatrix}$$

$$\frac{\partial \vec{v}}{\partial x} \begin{cases} \frac{\partial P}{\partial x} = y \\ \frac{\partial Q}{\partial x} = -2x \end{cases}$$

$$\frac{\partial \vec{v}}{\partial y} \begin{cases} \frac{\partial P}{\partial y} = x \\ \frac{\partial Q}{\partial y} = 2y \end{cases}$$

look at $(2,0)$, so

$$\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial Q}{\partial x} = -4$$

$$\frac{\partial P}{\partial y} = 2 \qquad \frac{\partial Q}{\partial y} = 0$$

$\frac{\partial P}{\partial x}$ } change in x-component as we move in the x

$\frac{\partial Q}{\partial x}$ } change in y-component as we move in the x

$\frac{\partial P}{\partial y}$ } change in ~~x~~ component as we move in the y

$\frac{\partial Q}{\partial y}$ } change in y-component as we move in the y

4 DIFFERENT POSSIBLE PARTIAL DERIVATIVES!!! (FOR VECTOR-VALUED FUNCTIONS)