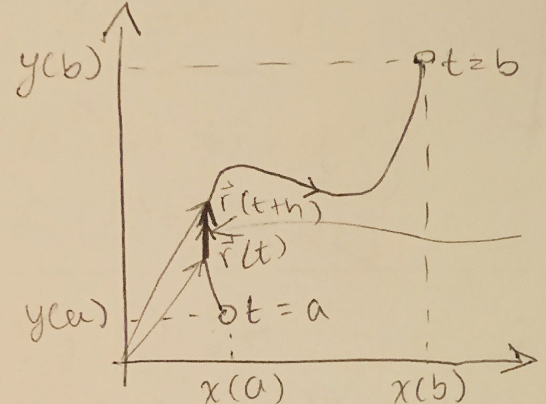
PARAMECTRICS

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$



how fast is r changing IRT t?

$\vec{r}(t+h) - \vec{r}(t) =$

$x(t+h)\hat{i} + y(t+h)\hat{j} - x(t)\hat{i} - y(t)\hat{j}$

$$\frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \hat{i}\,\frac{\left(x(t+h) - x(t)\right)}{h} + \hat{j}\,\frac{\left(y(t+h) - y(t)\right)}{h}$$

$$\underbrace{\lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}}_{r'(t)} = \underbrace{\lim_{h\to 0} \frac{(x(t+h) - x(t))\hat{i} + (y(t+h) - y(t))\hat{j}}{h}}_{x'(t)\hat{i} + y'(t)\hat{j}}$$

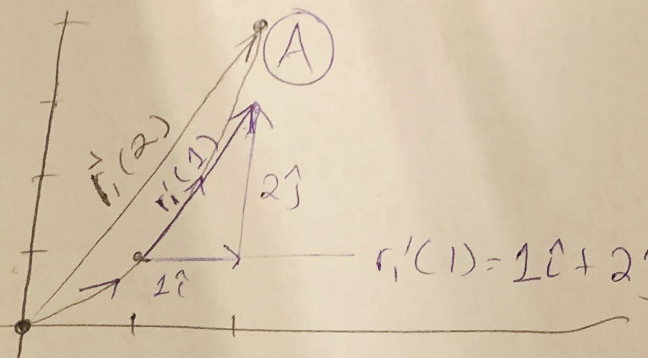
$$r'(t) = x'(t)\hat{i} + y'(t)\hat{j}$$

$$\left(\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right)dt$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$$

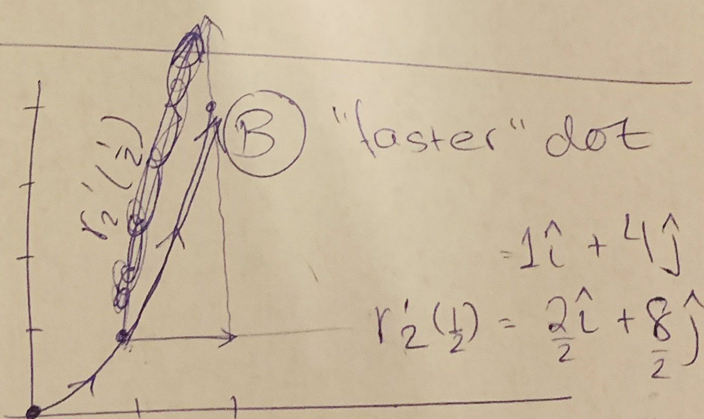
PARAMETRICS

$x_1(t) = t$
$y_1(t) = t^2$
$0 \le t \le 2$



$r_1(t) = t\hat{i} + t^2\hat{j}$
$r_1'(t) = 1\hat{i} + 2t\hat{j}$

$x_2(t) = 2t$
$y_2(t) = (2t)^2 = 4t^2$
$0 \le t \le 1$



$\vec{r}_2(t) = 2t\hat{i} + 4t^2\hat{j}$
$\vec{r}_2'(t) = 2\hat{i} + 8t\hat{j}$

Different parametrizations... but SAME PATH!

but... (A) takes 2 seconds to complete the path
but (B) takes 1 second to complete the path
so,... $\|\underbrace{r_2'(t)}\| > \|\underbrace{r_1'(t)}\|$, so r_2 is faster than r_1

velocity