

SOLUTIONS:

PARTIAL DERIVATIVES

Task:

$$\textcircled{1} f(x, y) = 3x + 4y$$

$$\frac{\partial f}{\partial x} = 3, \quad \frac{\partial f}{\partial y} = 4$$

$$\textcircled{2} f(x, y) = x e^{2x+3y}$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= 1(e^{2x+3y}) + x \cdot \frac{\partial f}{\partial x}(e^{2x+3y}) \\ &= e^{2x+3y} + x \cdot \frac{\partial f}{\partial x}(2x+3y) \cdot e^{2x+3y} \end{aligned}$$

$$\frac{\partial f}{\partial x} = e^{2x+3y} + 2x e^{2x+3y}$$

SOLUTIONS: PARTIAL DERIVATIVES

Joke:

$$(2) f(x, y) = x e^{2x+3y}$$

$$\frac{\partial f}{\partial y} = x \left(\frac{\partial f}{\partial y} e^{2x+3y} \right)$$

$$\frac{\partial f}{\partial y} = x (3 e^{2x+3y})$$

$$\therefore \frac{\partial f}{\partial y} = 3x e^{2x+3y}$$

SOLUTIONS:

PARTIAL DERIVATIVE $f \rightarrow$

Joke:

$$\textcircled{3} f(x, y) = \frac{x-y}{x+y}$$

$$\frac{\partial f}{\partial x} = \frac{1(x+y) - (1)(x-y)}{(x+y)^2}$$

$$\frac{\partial f}{\partial x} = \frac{x+y-x+y}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-1(x+y) - 1(x-y)}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-x-y-x+y}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

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SOLUTIONS: PARTIAL DERIVATIVES

Nice:

$$\textcircled{1} f(x, y) = 2x \sin(x^y y)$$

$$\frac{\partial f}{\partial x} = 2 \sin(x^y y) + 2x \frac{\partial}{\partial x} \sin(x^y y)$$

$$\frac{\partial f}{\partial x} = 2 \sin(x^y y) + 2x \cdot 2xy \cdot \cos(x^y y)$$

$$\frac{\partial f}{\partial x} = 2 \sin(x^y y) + 4x^2 y \cos(x^y y)$$

$$\frac{\partial f}{\partial y} = 2x \frac{\partial}{\partial y} (\sin(x^y y))$$

$$\frac{\partial f}{\partial y} = 2x \cdot x^y \cdot \cos(x^y y)$$

$$\frac{\partial f}{\partial y} = 2x^3 \cos(x^y y)$$

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Solutions:

PARTIAL DERIVATIVES

Hand:

$$\textcircled{1} f(x, y, z) = x \cos z + x^2 y^3 e^z$$

$$\frac{\partial f}{\partial x} = \cos z + 2x y^3 e^z$$

$$\frac{\partial f}{\partial y} = 3y^2 x^2 e^z$$

$$\frac{\partial f}{\partial z} = x(-\sin z) + x^2 y^3 e^z$$

$$\frac{\partial f}{\partial z} = -(\sin z)x + x^2 y^3 e^z$$

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