PARTIAL DERIVATIVES

$f(x,y) = x^2y + \sin y$

GENERAL:

$$\underbrace{\frac{\partial f}{\partial x}(x,y)}_{\text{partial } f \text{ WRT partial } x \text{ evaluated at } (x,y)} = \underbrace{\frac{\partial}{\partial x}(x^2y + \sin(y))}_{\text{treat } y \text{ like a CONSTANT!}}$$

$$\frac{\partial f}{\partial x} = 2xy + 0$$

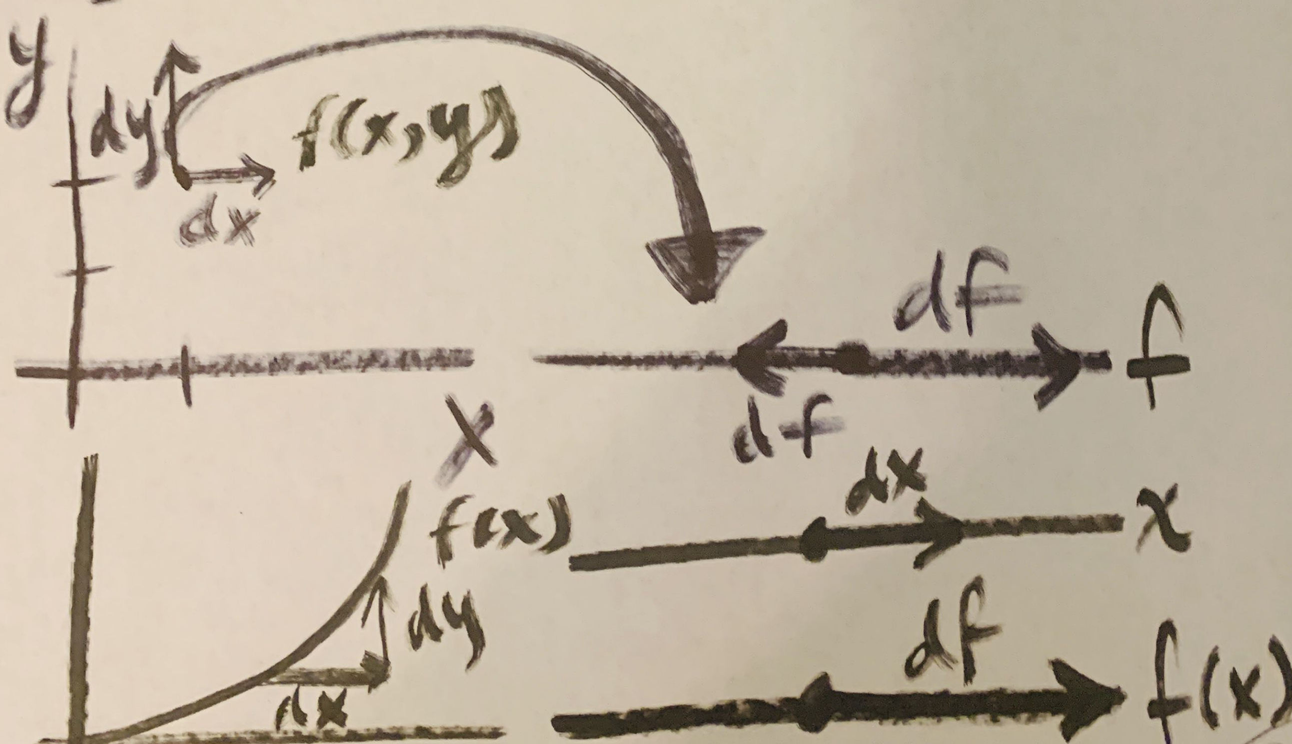
$$\frac{\partial f}{\partial y} = x^2 + \cos y$$

PARTAL DERIVATIVES

SPECIFIC:

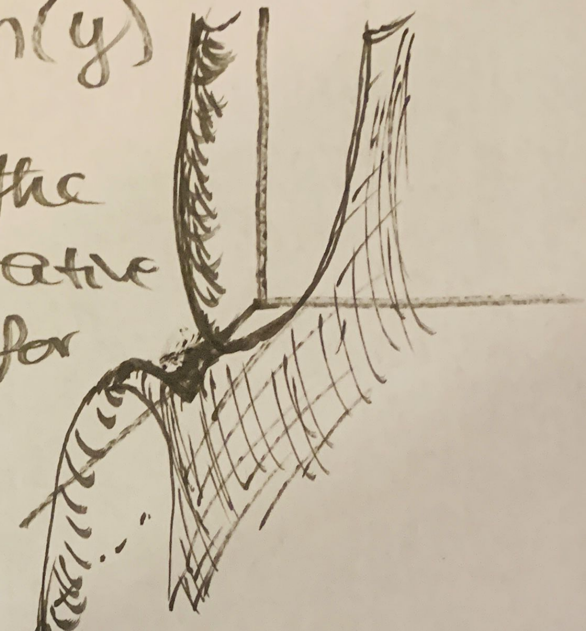
$$\frac{\partial f}{\partial x}(1,2) = 2(1)(2) = 4$$

$$\frac{\partial f}{\partial y}(1,2) = (1)^2 + \cos(2)$$



PARTIAL DERIVATIVES

$f(x,y) = x^2y + \sin(y)$

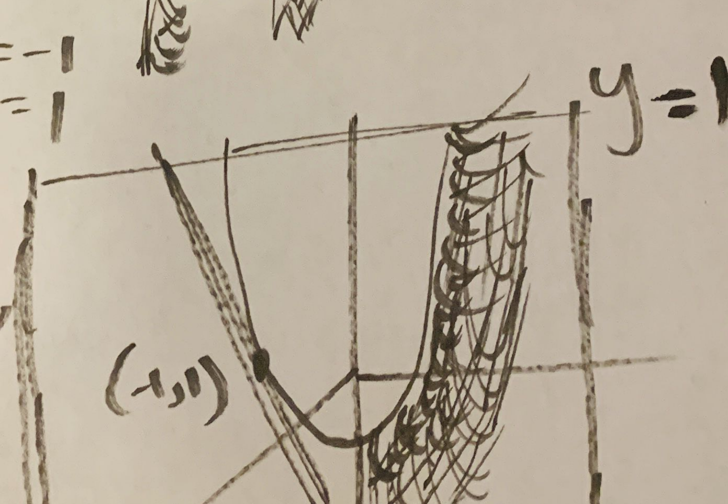


Q: What does the partial derivative actually mean for the graph?

$$\frac{\partial f}{\partial x}(-1,1) = 2xy \Big|_{\substack{x=-1 \\ y=1}}$$

$$= 2(-1)(1)$$

$$= -2$$



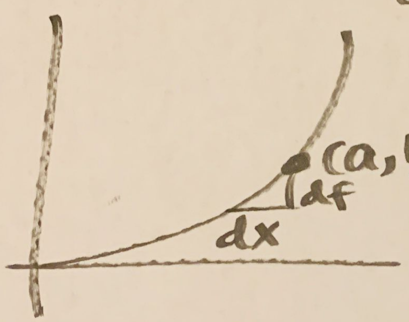
treating y as a constant "slices" the graph at that y and THEN finds the slope

PARTIAL DERIVATIVES

The formal definition of $\frac{\partial f}{\partial x}$:

> Let's begin by looking at the SINGLE-VARIABLE analogue:

$$f(x) = x^2 \rightarrow \frac{df}{dx}(a,b)$$



$$\underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{\text{limit definition of the derivative}}$$

SINGLE VARIABLE: $\cancel{\frac{\partial f}{\partial}}\ \frac{df}{dx}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

MULTI VARIABLE: $\cancel{\frac{\partial f}{\partial x}(a,b) = \lim}$

$$\frac{\partial f}{\partial x}(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

$$\frac{\partial f}{\partial y}(a,b) = \lim_{h \to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

PARTIAL DERIVATIVES

Symmetry of second partial derivatives

$$f(x,y) = \sin(x)\,y^2$$

$\frac{\partial f}{\partial x}$: $\cos(x)y^2$

$\frac{\partial f}{\partial y}$: $2\sin(x)\,y$

$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$: $-\sin(x)y^2$

$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$: $2y\cos x$

$\cancel{\frac{\partial^2 f}{\partial y^2}}$

$\frac{\partial f}{\partial x}$: $2y\cos(x) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$

$\left(\frac{\partial f}{\partial y}\right)\frac{\partial}{\partial y} = \frac{\partial^2 f}{\partial y^2}$: $2\sin(x)$

$$\boxed{\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}}$$

$$\boxed{\frac{\partial^2 f}{\partial y \partial x} = f_{xy}}$$