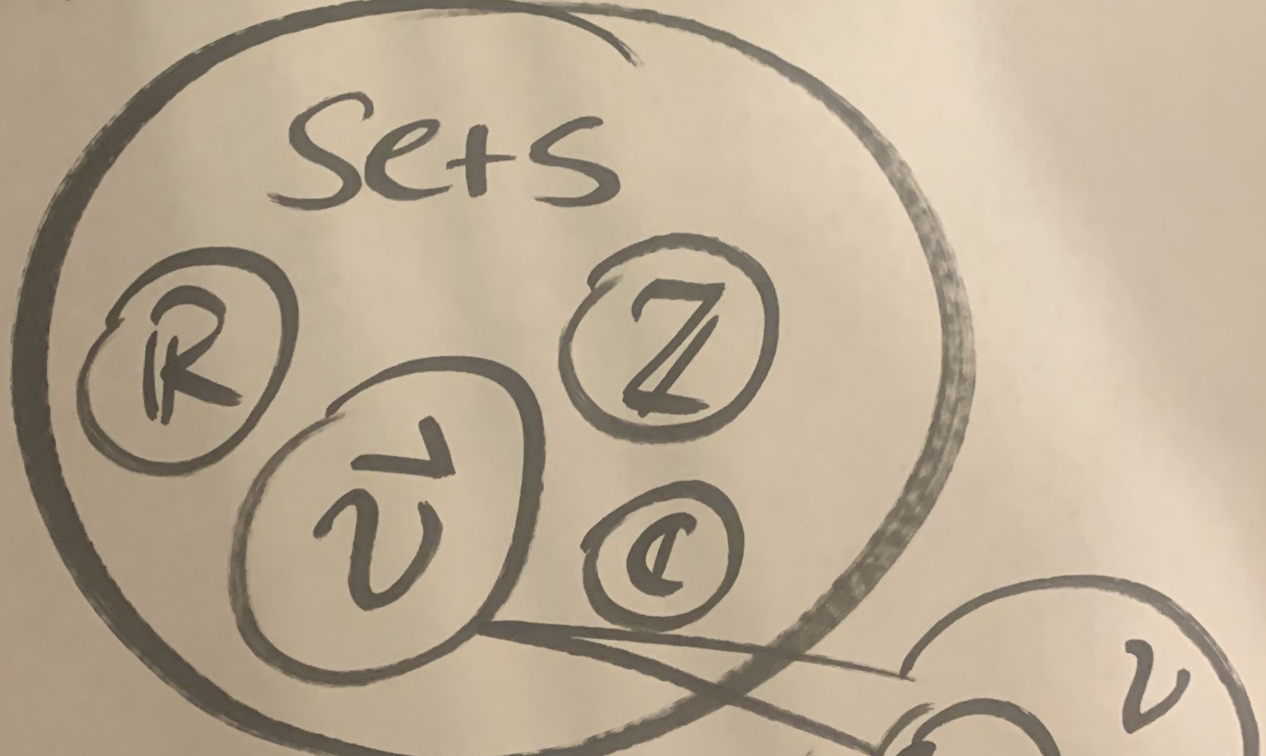
Tensor: RULES

$\{VS\}$ all elements of this set are vectors

vector space



{VS} must be closed under...

if $w + v = t$

where $w, v \in VS$

then $t \in VS$

$\boxed{\mathbb{R}}$ — a

scalars

$\underbrace{aw \in VS}_{\text{real vector space}}$

$\boxed{\mathbb{C}}$ — a

$aw \in VS$

$$\boxed{\begin{matrix} v & + & * \\ a & \in & \mathbb{R} \end{matrix}}$$

LINEAR COMBINATION	$a\vec{w}+b\vec{z}=\vec{k}$
LINEARITY	$\cancel{a\vec{w}+b\vec{z}}$ $a\vec{w}+a\vec{z}=$ $a(\vec{w}+\vec{z})$

* EVERY VECTOR SPACE HAS 0, b/c if

$w \in VS$,

$-w \in VS$

$\overline{w-w \in VS} \Rightarrow 0 \in VS$

$\boxed{V \mid +,\ * \mid a \in \mathbb{R}}$ $\boxed{W \mid +,\ * \mid a \in \mathbb{R}}$

$w \quad q \quad p \qquad r \quad s \quad t$

$v + q = m \qquad r + s = t$

$\nexists\, w + r$

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basis vector of a vector space... basis vectors aren't unique... but the dimensions are!

TWO VECTOR SPACES ARE ISOMORPHIC IF YOU CAN ESTABLISH A 1-1 CORRESPOND-ENCE B'TWN. THEM & OPERATIONS IN BOTH V.S. ARE IDENTICAL

$v, w \in \mathbb{R}$

$v + w$

$\cancel{v \cdot w \in \mathbb{R}}$

$\cancel{v \times w \in \mathbb{R}}$

$\cancel{\|v\|^2 = v \cdot v}$

$\boxed{V \quad 4 \quad \mathbb{R}}$

dimen-sionality

6

$n = aw + bv + ty' + d\ell$

~~Of~~ assume $\exists\, e_\mu$

$\underbrace{e_\mu}_{\text{set of basis vectors}}$

$\mathcal{V}$ (4) $\mathbb{R}$

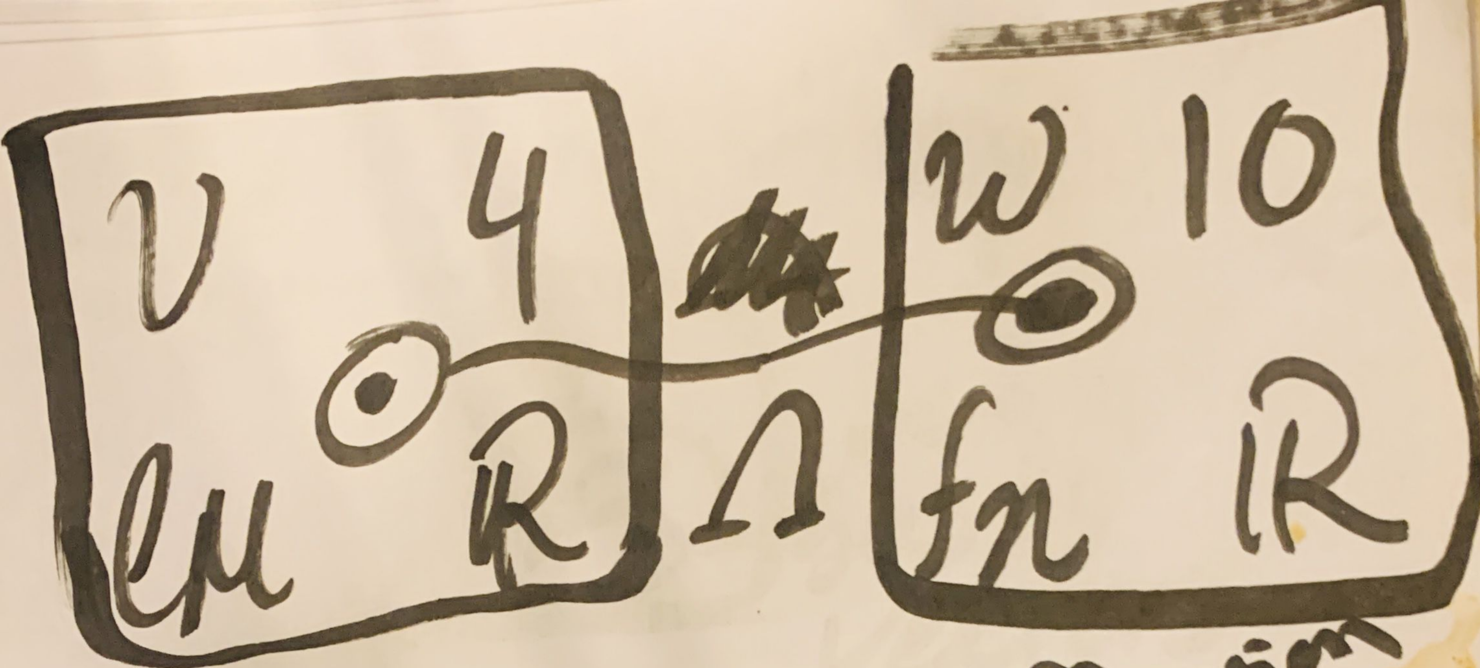
where $\mu = 0, 1, 2, 3, 4$:

$e_\mu = \{e_0, e_1, e_2, e_3\}$

$$A = \vec{A} = a e_0 + b e_1 + c e_2 + d e_3$$

$$= A^0 e_0 + A^1 e_1 + A^2 e_2 + A^3 e_3$$

Einstein Summation convention $\Big[= A^\mu e_\mu = \sum_{\mu=0}^{4} A^\mu e_\mu$



$\Lambda : V \to W$ domain range notation

$\Lambda V \to W$ operator notation

$\Lambda(V) \to W$ function notation

$\langle \Lambda, V \rangle \to W$ braket notation

If I can find ~~the~~ where e_μ maps to from V to K, I'm

HOME FREE!!!

$$\Omega e_0 = \langle \Omega, e_0 \rangle = \cancel{\exists f_1 + \dots}\ \cancel{A}B^{\mu n}_{fn}$$

$$\Omega e_1 = \langle \Omega, e_1 \rangle = C^{\mu n}_{fn}$$

$$\Omega e_2 = \langle \Omega, e_2 \rangle = D^{\mu n}_{fn}$$

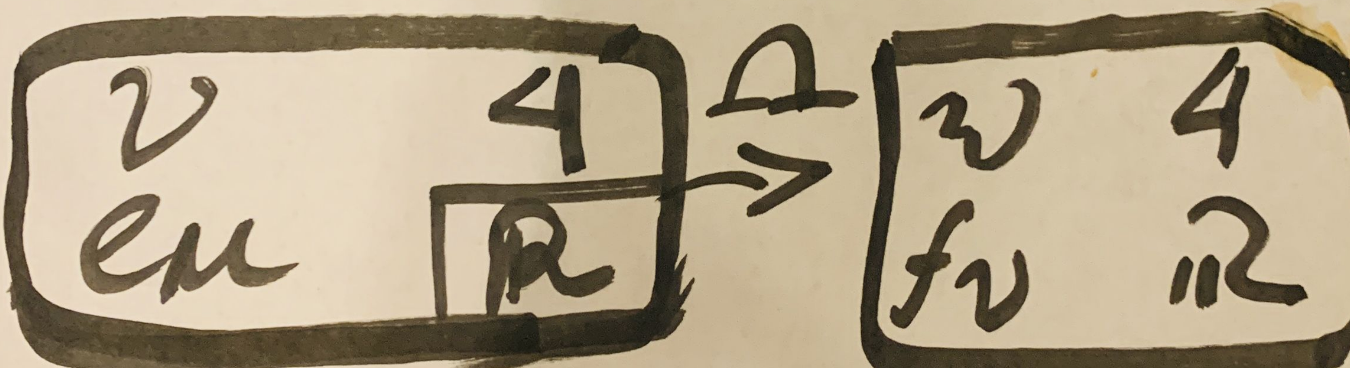
$$\Omega e_3 = \langle \Omega, e_3 \rangle = E^{\mu n}_{fn}$$

LINEAR MAPS!

$$\langle \Lambda, A \rangle = \langle \Lambda, A^{\mu} e_{\mu} \rangle$$

$$= A^{0}\langle \Lambda, e_{0} \rangle + A^{1}\langle \Lambda, e_{1} \rangle + A^{2}\underbrace{\langle \Lambda, e_{2} \rangle} + A^{3}\langle \Lambda, e_{3} \rangle$$

$$\langle \Lambda, e_{2} \rangle = \beta^{\nu} f_{\nu}$$



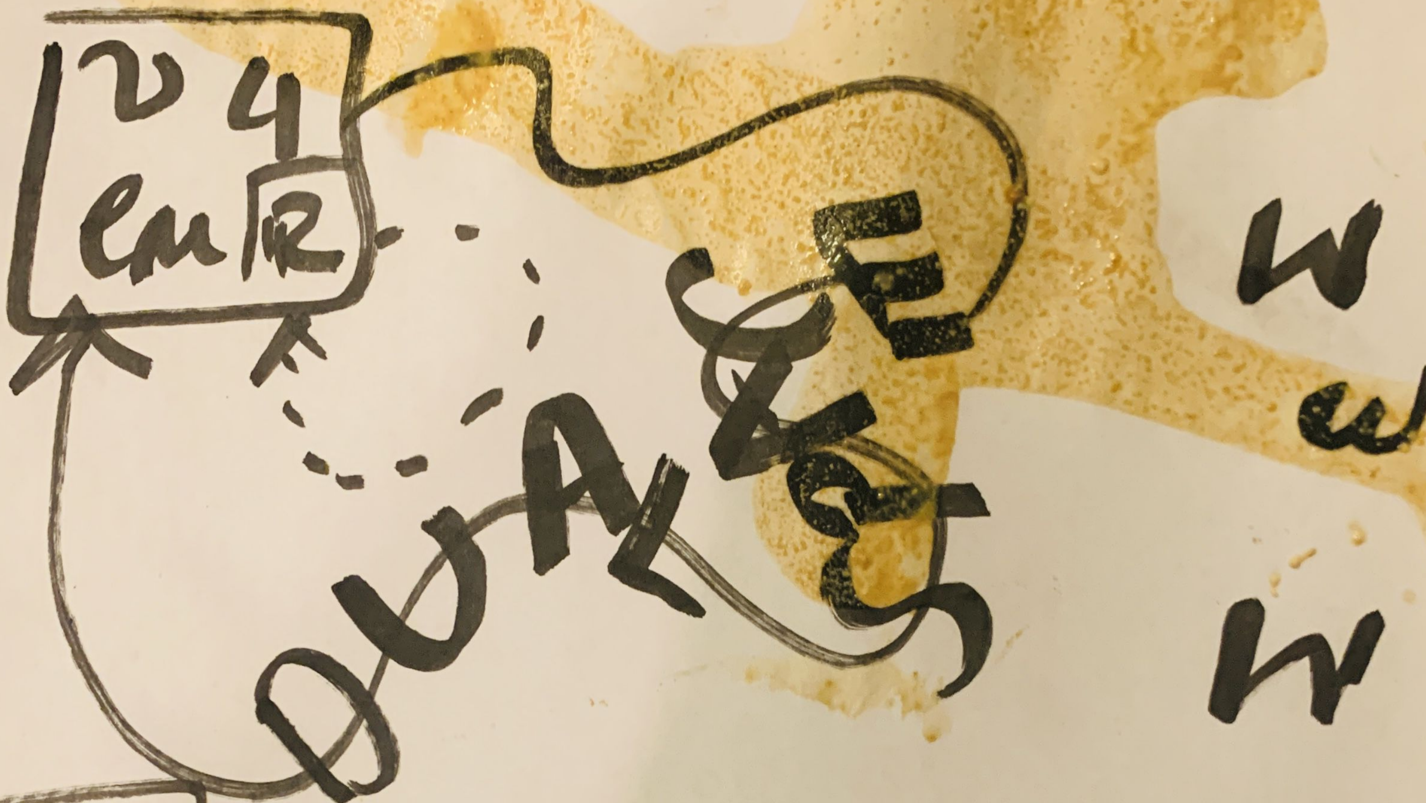
$\Lambda e_0 = |$

$\Lambda e_1 = ||$

$\Lambda e_2 = |||$

$\Lambda e_3 = ||||$

$$\langle \Lambda, A^{\mu} e_{\mu} \rangle = A^{0}\langle \Lambda, e_{0} \rangle + A^{1}\langle \Lambda, e_{1} \rangle + A^{2}\langle \Lambda, e_{2} \rangle + A^{3}\langle \Lambda, e_{3} \rangle$$



$\boxed{\mathbb{R}}$ is a 1-D vector space

realization of 1-D vector space

NO NEED for another vector space! just map into $\mathbb{R}$

$$\boxed{\begin{matrix} v & \\ \ell_\mu & \mathbb{R}^4 \end{matrix}} \xrightarrow{\Omega} \boxed{\mathbb{R}}$$

consider the set of

ALL POSSIBLE MAPS!

dual space:

$V^* : V \to \mathbb{R}$

a vector space!

TB

LINEAR MAP.

$$\langle \Lambda, v + p \rangle = \langle \Lambda, v \rangle + \langle \Lambda, p \rangle$$

$$\langle \Lambda, a v + b p \rangle = a \langle \Lambda, v \rangle + b \langle \Lambda, p \rangle$$

$$\boxed{V} \xrightarrow{\Lambda} \boxed{W}$$

given any $A \in V$,
$?? \in W$?

well... $A = A^{\mu} e_{\mu}$