1 Finding λ and f

1. Microwave oven operating at a wavelength of 12.2 cm.

   Since the speed of light is constant and $v = f\lambda$, we have

$$f = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{12.2 \text{ cm} * 10^{-2} \frac{\text{m}}{\text{cm}}} = \boxed{2.459 * 10^9 \text{ Hz}}$$

2. The $9.19263177 * 10^9$ Hz transition in cesium atom.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{9.19263177 * 10^9 \text{ Hz}} = \boxed{0.0326349455 \text{ m}}$$

3. The peak of the radiation emitted by the human body near 10μm

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{10\mu\text{m} * 10^{-6} \frac{\text{m}}{\mu\text{m}}} = \boxed{3 * 10^{13} \text{ Hz}}$$

4. Optical communication wavelength of 1.55 μm.

$$f = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{1.55\mu\text{m} * 10^{-6} \frac{\text{m}}{\mu\text{m}}} = \boxed{1.93548387 * 10^{14} \text{ Hz}}$$

5. National Public Radio broadcasting at 93.9 MHz.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{93.9 \text{ MHz} * 10^6 \frac{\text{Hz}}{\text{MHz}}} = \boxed{3.1948 \text{ m}}$$

6. A.M. radio operating at 880 kHz.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{880 \text{ KHz} * 10^3 \frac{\text{Hz}}{\text{KHz}}} = \boxed{340.9090 \text{ m}}$$

7. Video Carrier frequency of 503.25 MHz for TV.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{503.25 \text{ MHz} * 10^6 \frac{\text{Hz}}{\text{MHz}}} = \boxed{0.5961 \text{ m}}$$

8. X-rays of frequency $7 * 10^{18}$ Hz used in medical imaging.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{7 * 10^{18} \text{ Hz}} = \boxed{4.28 * 10^{-11} \text{ m}}$$

9. A gamma ray of wavelength 55 fm.

$$f = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{55 \text{ fm} * 10^{-15} \frac{\text{m}}{\text{fm}}} = \boxed{5.4545 * 10^{21} \text{ Hz}}$$

10. He-Ne laser light of wavelength 632.8 nm used in barcode scanning.

$$f = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{632.8 \text{ nm} * 10^{-9} \frac{\text{m}}{\text{nm}}} = \boxed{1 * 10^9 \text{ Hz}}$$

11. Terahertz radiation (T-ray) of frequency 2.5 THz.

$$\lambda = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{2.5 * 10^{12} \text{ Hz}} = \boxed{1.2 * 10^{-4} \text{ m}}$$

12. The CMB radiation of wavelength 7.35 cm

$$f = \frac{3 * 10^8 \frac{\text{m}}{\text{s}}}{7.35\text{cm} * 10^{-2} \frac{\text{m}}{\text{cm}}} = \boxed{4.08 * 10^9 \text{ Hz}}$$

2 Equivalent Solutions to Wave Equation

1. $\psi(x,t) = A \sin k(x \pm vt)$

   The wave equation, $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 x}{\partial x^2}$, has solutions of the form $u = u(x \pm vt)$. This equation, with the k distributed throughout, becomes $\psi(x,t) = A \sin(kx \pm kvt)$.

2. $\psi(x,t) = A \sin(kx \pm \omega t)$

   We recall that $\frac{\omega}{k} = v$, a fact that is clear both from dimensional analysis and derivation:

$$\frac{\frac{1}{s}}{\frac{1}{m}} = \frac{m}{s} \text{ or } \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda = v$$

   Factoring k from the phase argument, we thus have

$$\psi(x,t) = A \sin k(x \pm \frac{\omega}{k} t) \to \boxed{\psi(x,t) = A \sin k(x \pm vt)}$$

3. $\psi(x,t) = A \sin 2\pi\{(x/\lambda) \pm (t/T)\}$

   Recall how ω, k relate to T, λ:

$$\omega = 2\pi f = \frac{2\pi}{T} \to T = \frac{2\pi}{\omega} \text{ and } k = \frac{2\pi}{\lambda} \to \lambda = \frac{2\pi}{k}$$

   Substituting the relevant terms, we have

$$\psi(x,t) = A \sin 2\pi\{(x/(\frac{2\pi}{k})) \pm (t/(\frac{2\pi}{\omega}))\}$$

$$\psi(x,t) = A \sin 2\pi\{\frac{kx}{2\pi} \pm \frac{\omega t}{2\pi}\} \to \boxed{\psi(x,t) = A \sin(kx \pm \omega t)}$$

   This was shown to be equivalent to $\psi(x,t) = A \sin k(x \pm vt)$ in the previous problem.

4. $\psi(x,t) = A \sin 2\pi f\{(x/v) \pm t\}$

   We recognize that $2\pi f = \omega$, which gives

$$\psi(x,t) = A \sin \omega\{(x/v) \pm t\} = A \sin \omega\{\frac{x}{(\frac{\omega}{k})} \pm t\} = A \sin \omega\{\frac{kx}{\omega} \pm t\}$$

$$\boxed{\psi(x,t) = A \sin(kx \pm \omega t)}$$

3 Traveling Wave

A traveling wave is one which may be expressed as $e^{-a(x\pm vt)^2}$. Why? e^{-x^2}[1] looks like a wave, but to make it move, we replace x with $x \pm vt$. We now recognize that the exponential may be factored as

$$5e^{(-ax^2-bt^2-2(ab)^{\frac{1}{2}}xt)} = 5e^{-(ax^2+2\sqrt{ab}xt+bt^2)} = 5e^{-(x\sqrt{a}+t\sqrt{b})^2} = 5e^{-(\sqrt{a}(x+\sqrt{\frac{b}{a}}t))^2} = 5e^{-a(x+\sqrt{\frac{b}{a}}t)^2}$$

We recognize that the velocity is simply the coefficient of the t term, which is $v = \sqrt{\frac{b}{a}}$. In this case, $a = 25\text{m}^{-1}, b = 9\text{s}^{-1}$. We thus have a traveling wave with velocity $\sqrt{\frac{25}{9}} = \boxed{\frac{5}{3}\frac{\text{m}}{\text{s}}}$ in the negative direction (owing to the positive vt term). Below is a sketch of the wave at $t = 0$. A few computed valuees of $f(x,t)$ are as follows: $f(0,0) = 5, f(0,0.5) = 0.526, f(0.1,0.6) = 0.0252$.

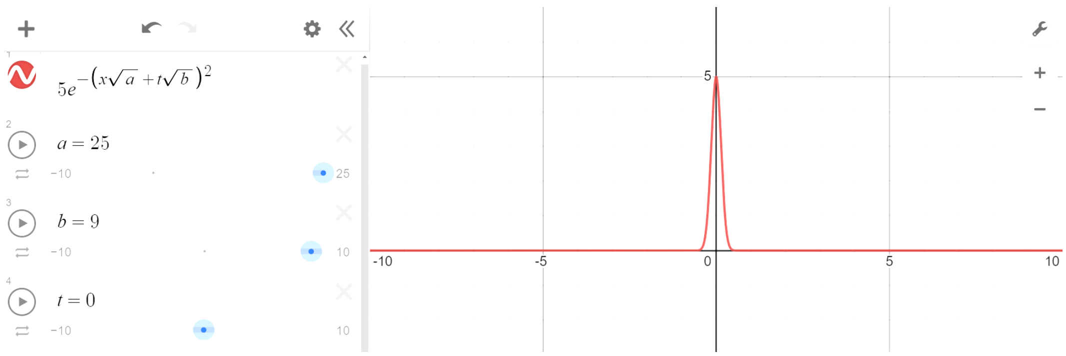

Figure 1: Wave $f(x,t) = f(x,0)$

4 Time-Averaged Values of $\sin^2(x)$ and $\cos^2(x)$

1. Prove $\langle \sin^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = \frac{1}{2}$

   Per the definition of the time-averaged function $f(t)$, we have

$$\frac{1}{T}\int_t^{t+T} \sin^2\left(\vec{k}\cdot\vec{r} - \omega t'\right) dt'$$

We now make a u-substitution

$$u = (k_x x + k_y y + k_z z) - wt' \to \frac{1}{T}\int_t^{t+T} \sin^2(u)\frac{du}{-w}$$

Leveraging the cosine double angle formula, we now have

$$-\frac{1}{wT}\left[\frac{1}{2}\left(\vec{K}\cdot\vec{r} - wt'\right) - \frac{1}{4}\sin\left(2\left(\vec{k} + \bar{r} - wt'\right)\right)\right]_{t'=t}^{t'=t+T}$$

[1] Coincidentally, it is also the normal distribution.

If we now let $p = 2(\vec{k} \cdot \vec{r} - 2\omega t)$, we have

$$\frac{1}{2} - \frac{1}{2\omega T}\left\{\left[-\frac{1}{2}\sin(p - 2\omega T) + \frac{1}{2}\sin(p)\right]\right\}$$

Expanding via the sine of a difference, everything inside the brackets becomes 0 to give

$$\frac{1}{2} - \frac{1}{4\omega T}\left\{[-\sin(p)\cos(2\omega T) - \sin(2\omega T)\cos(p) + \sin(p)]\right\} = \boxed{\frac{1}{2}}$$

2. Prove $\langle \cos^2(\vec{k} \cdot \vec{r} - \omega t)\rangle = \frac{1}{2}$

   Recall that by definition, a time-averaged function $f(t)$ has

$$\langle f(t)\rangle = \frac{1}{T}\int_t^{t+T} f(t')dt'$$

   For $f(t) = \cos^2(\cdot)$, we have

$$\frac{1}{T}\int_t^{t+T} \cos^2(\vec{k} \cdot \vec{r} - \omega t')dt'$$

   We proceed to make the u-substitution

$$u = \vec{k} \cdot \vec{r} - \omega t \rightarrow \frac{du}{dt} = -\omega \rightarrow -\frac{du}{w} = dt$$

   This produces the integral

$$-\frac{1}{\omega T}\int \cos^2(u)du$$

   Leveraging the trigonometric formula for cosine that $\cos^2(u) = \cos^2(u) - \sin^2(u)$, which may be rewritten using $\sin^2 u = 1 - \cos^2 u$. We thus have the integral

$$-\frac{1}{2\omega T}\int \cos(2u) + 1du = -\frac{1}{2\omega T}[\frac{\sin(2u))}{2} + u]$$

   Expanding by substituing the limits of integration, we have

$$-\frac{1}{\omega T}[\frac{\vec{k} \cdot \vec{r} - \omega t - \omega T}{2} + \frac{\sin(2(\vec{k} \cdot \vec{r}) - 2\omega t - 2wT)}{4} - (\frac{\vec{k} \cdot \vec{r} - \omega t - \omega T}{2}) - (\frac{\sin(2(\vec{k} \cdot \vec{r}) - 2\omega T)}{4})]$$

   Simplifying by letting $p = 2(\vec{k} \cdot \vec{r}) - 2\omega t$, we have

$$\frac{1}{2} - \frac{1}{\omega T}[\frac{\sin(p - 2\omega T) - \sin(p)}{4}] = \boxed{\frac{1}{2}}$$

3. Prove $\langle \sin(\vec{k}\cdot\vec{r}-\omega t)\cos(\vec{k}\cdot\vec{r}-\omega t)\rangle = 0$

   We exploit the trigonometric identity

$$\sin(2x) = 2\sin(x)\cos(x)$$

   This enables us to rewrite our time-averaged value as

$$\langle \sin(\vec{k}\cdot\vec{r}-\omega t)\cos(\vec{k}\cdot\vec{r}-\omega t)\rangle = \frac{1}{2T}\int_t^{t+T} \sin(2(\vec{k}\cdot\vec{r}-\omega t'))dt'$$

   We now use u-substitution as

$$u = 2(\vec{k}\cdot\vec{r}) - 2wt' \rightarrow -\frac{du}{2w} = dt'$$

   Notwithstanding the limits of integration, we may now re-write the integral as

$$-\frac{1}{4\omega T}\int \sin(u)du = \frac{1}{4\omega T}\cos(u) = \frac{1}{4\omega T}\cos(2(\vec{k}\cdot\vec{r}) - 2wt')\Bigg|_{t'=t}^{t'=t+T}$$

   Substituting the limits of integration gives

$$\frac{1}{4\omega T}[\cos(2(\vec{k}\cdot\vec{r}) - 2wt - 2wT) - \cos(2(\vec{k}\cdot\vec{r}) - 2wt)]$$

   Let $p = 2(\vec{k}\cdot\vec{r}) - 2wt$, as this term is repeated:

$$\frac{1}{4\omega T}[\cos(p - 2\omega T) - \cos(p)]$$

   Expanding the cosine of a difference, we have

$$\frac{1}{4\omega T}[\cos(p)\cos(2\omega T) + sin(p)\sin(2\omega T) - \cos(p)]$$

   As $T = n\tau$, where τ is the period, we have $\sin(2\omega T) = 0$ and $\cos(2\omega T) = 1$, which gives

$$\frac{1}{4\omega T}[\cos(p) - \cos(p)] = \boxed{0}$$

5 Properties of an Electromagnetic Wave

1. Find direction of propagation.

   We begin with the wave given by

$$\vec{E}(\vec{r},t) = [2\hat{i} - 3\hat{j} + 2\hat{k}](10^4)\cos(5*10^7(\frac{x+2+2z}{3}) - (15\pi)*10^{15}t)$$

We can now read off $\vec{k}$ from the phase argument as

$$k_x = \frac{1}{3}5 * 10^7, k_y = \frac{2}{3}5 * 10^7, k_z = \frac{2}{3}5 * 10^7$$

The magnitude of $\vec{k}$ is thus

$$\|\vec{k}\| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \frac{\pi}{2} * 10^8 \mathrm{m}^{-1}$$

The direction of propagation may be found by taking the unit vector $\hat{s}$ as

$$\boxed{\hat{s} = \frac{k}{\|k\|} = \frac{1}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}}$$

2. Find the velocity.

Recall that the velocity of a wave is given by

$$v = \frac{\omega}{k} = \frac{15\pi * 10^{15}}{(\frac{\pi}{2} * 10^8)} = \boxed{3 * 10^8 \mathrm{m/s}}$$

3. Find the wavelength

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2} * 10^8} = \boxed{4 * 10^{-8} m}$$

4. Find $\vec{B}(\vec{r}, t)$

$$k \times E = \omega B \to B = \frac{k \times E}{\omega} = \frac{\|k\| \hat{s} \times \vec{E}}{\omega}$$

We now substitute $\hat{s}$ from part one, E as given, and ω from the time term.

$$\frac{\frac{\pi}{2} * 10^8}{15\pi * 10^{15}} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 2 & -3 & 2 \end{vmatrix} (10^4) \cos\left(5 * 10^7 \left(\frac{x + 2y + 2z}{3}\right) - (15\pi * 10^{15})t\right)$$

$$\boxed{\vec{B}(\vec{r}, t) = \left[\frac{7}{9}\hat{\imath} - \frac{7}{9}\hat{k}\right] (10^{-4}) \cos\left(5 * 10^7 \left(\frac{x + 2y + 2z}{3}\right) - (15\pi * 10^{15})t\right)}$$

5. Find the intensity.

We simply have to plug in our electric field's maximum amplitude into the following:

$$S_{avg} = \frac{1}{2\mu_0} EB = \frac{1}{2}(4\pi{*}10^{-7}) \left(\sqrt{10^4((2)^2 + (-3)^2 + (2)^2)}\right)^2 \left(\sqrt{10^{-4}\left(\left(\frac{7}{9}\right)^2 + \left(-\frac{7}{9}\right)^2\right)}\right)^2$$

This gives $\boxed{I = S_{avg} = 1.29 * 10^{-5} \frac{\mathrm{W}}{\mathrm{m}^2}}$

6. Describe the polarization state of the wave.

The wave is linearly polarized because E_0 is constant and time-invariant. The direction of polarization is given by the unit vector

$$\hat{e} = \frac{\vec{E_0}}{\|\vec{E_0}\|} = \frac{[2\hat{i} - 3\hat{j} + 2\hat{k}]}{\sqrt{13}} = \boxed{\frac{2}{\sqrt{13}}\hat{i} - \frac{3}{\sqrt{13}}\hat{j} + \frac{2}{\sqrt{13}}\hat{k}}$$

7. Find the radiation pressure for a perfectly absorbing surface.

For a perfectly absorbing surface, simply substitute values into

$$\rho = \frac{I}{c} = \frac{1.29 * 10^{-5}}{3 * 10^8} = \boxed{0.43 * 10^{-13}\frac{\text{N}}{\text{m}^2}}$$

6 Radar System

A surveillance radar system operating at 12 GHz at 180 kW of power, attempts to detect an incoming stealth aircraft at 90 km. Assume the radar beam is emitted uniformly over a hemisphere.

1. What is the intensity of the beam when it reaches the aircraft's location?

We know that intensity obeys an inverse square law, so that it is initially simply the power, but at a distance of 90 km, it becomes $I = \frac{P}{4\pi r^2} = \frac{180000}{4\pi(90000)^2} = \boxed{1.76 * 10^{-6}\frac{\text{W}}{\text{m}^2}}$

2. What is the power of the reflected beam if the aircraft reflects the beam with a cross sectional area of 0.22 m^2.

Recall that the power is given by $I = \frac{P}{A} \to P = IA$. We simply take the intensity from the previous part and multiply by the cross-sectional area of the aircraft:

$$P = \left(1.76 * 10^{-6}\frac{\text{W}}{\text{m}^2}\right)(0.22\text{m}^2) = \boxed{3.872 * 10^{-5}\text{W}}$$

3. Back at the radar site, what is the intensity?

Consider now that the airplane is the source and the radar system is the receiver. We thus simply begin with the power found in the second part and drop that power as an inverse square law. We thus have $I' = \frac{P'}{4\pi r^2} = \frac{3.872 * 10^{-5}}{4\pi(90000)^2} = \boxed{3.8 * 10^{-16}\frac{\text{W}}{\text{m}^2}}$

4. Determine the maximum value of the electric field vector.

Recall that the intensity of radiation is related to the electric field's amplitude by

$$I = \frac{1}{c\mu_0}E_{rms}^2 = \frac{1}{c\mu_0}\left(\frac{E_0}{\sqrt{2}}\right)^2 \quad E_0 = \sqrt{2Ic\mu_0}$$

$$E_0 = \sqrt{2(3.8 * 10^{-16})(3 * 10^8)(4\pi * 10^{-7})} = \boxed{5.35 * 10^{-7}\frac{\text{V}}{\text{m}}}$$

5. Find the rms value of the magnetic field of the reflected radar beam.

Recall that the magnetic field is related to the intensity of the radiation by

$$I = \frac{E_0 B_0}{2\mu_0} \rightarrow B_0 = \frac{2\mu_0 I}{E_0} = \frac{2(4\pi * 10^{-7})(3.8 * 10^{-16})}{5.35 * 10^{-7}} = 1.78 * 10^{-15} \text{ T}$$

$$B_{rms} = \frac{B_0}{\sqrt{2}} = \frac{1.78 * 10^{-15}}{\sqrt{2}} = \boxed{1.26 * 10^{-15} \text{ T}}$$

7 Proof of $\vec{k} \cdot \vec{B} = 0$ & $(v/k)(\vec{k} \times \vec{B}) = -\vec{E}$

1. Prove that $\vec{k} \cdot \vec{B} = 0$

We begin with the differential form of Maxwell's second equation

$$\nabla \cdot \vec{B} = 0$$

Note that we may express the magnetic field in the form

$$\vec{B} = \vec{B_0} e^{i(\vec{k}\cdot\vec{r} - \omega t)} = (B_{x_0}\hat{i} + B_{y_0}\hat{i} + B_{z_0}\hat{k})e^{i(k_x x + k_y y + k_z z - \omega t)}$$

We now apply the divergence operator to $\vec{B}$ to find

$$\frac{\partial B_x}{\partial x} = \frac{\partial}{\partial x}(B_{x_0} e^{ip})$$

Where $p = \vec{k} \cdot \vec{r} - \omega t$. We thus have

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial p}\frac{\partial p}{\partial x} = i k_x B_{x_0} e^{ip} = i k_x B_x$$

Likewise for the other partial derivatives, we have

$$\frac{\partial B_y}{\partial y} = i k_y B_y, \frac{\partial B_z}{\partial z} = i k_z B_z$$

We thus have the divergence to be

$$\nabla \cdot \vec{B} = i k_x B_x + i k_y B_y + i k_z B_z = i(\vec{k} \cdot \vec{B}) = 0 \rightarrow \boxed{\vec{k} \cdot \vec{B} = 0}$$

2. Prove that $(v/k)(\vec{k} \times \vec{B}) = -\vec{E}$

We begin by considering the cross product $\nabla \times \vec{B}$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\left(\frac{\partial}{\partial y}B_x - \frac{\partial}{\partial x}B_y\right) + \hat{j}\left(\frac{\partial}{\partial x}B_z - \frac{\partial}{\partial z}B_x\right) + \hat{k}\left(\frac{\partial}{\partial x}B_y - \frac{\partial}{\partial y}B_x\right)$$

We now expand the partial derivative terms by considering the magnetic field as

$$\vec{B} = \vec{B_0} e^{i(k_x x + k_y y + k_z z) - i\omega t}$$
$$\vec{B_z} = B_{0_z} e^{i(k_x x + k_y y + k_z z) - i\omega t}$$
$$\frac{\partial B_z}{\partial y} = ik_y B_{0_z} e^{i(k_x x + k_y y + k_z z) - i\omega t} = ik_y B_{0_z} e^{ip} = ik_y B_z$$
$$\frac{\partial B_y}{\partial z} = ik_z B_{0_y} e^{i(k_x x + k_y y + k_z z) - i\omega t} = ik_z B_{0_y} e^{ip} = ik_z B_y$$

The first term of our cross product is then

$$\hat{i}\left(\frac{\partial}{\partial y} B_x - \frac{\partial}{\partial x} B_y\right) = \hat{i}(ik_y B_z - ik_z B_y)$$

In a similar manner, we may expand the other other two terms to produce

$$\hat{j}\left(\frac{\partial}{\partial x} B_z - \frac{\partial}{\partial z} B_x\right) = \hat{j}(ik_z B_x - ik_x B_z)$$

$$\hat{k}\left(\frac{\partial}{\partial x} B_z - \frac{\partial}{\partial z} B_x\right) = \hat{k}(ik_x B_y - ik_y B_x)$$

We may combine all these terms to obtain

$$\nabla \times \vec{B} = i[\hat{i}(ik_y B_z - ik_z B_y) + \hat{j}(k_z B_x - k_x B_z) + \hat{k}(k_x B_y - k_y B_x)]$$

The trained eye will recognize this as none other than the cross product $i(\vec{k} \times \vec{B})$. We now recall that $\nabla \times \vec{B} = \frac{1}{c^2}\frac{\partial E}{\partial t}$, which is simply Ampere's law in free space.

$$\vec{E} = \vec{E_0} e^{(k_x x + k_y y + k_z z) - i\omega t} \rightarrow \frac{1}{c^2}\frac{\partial E}{\partial t} = \frac{1}{c^2} - i\omega \vec{E} \rightarrow \frac{c^2}{\omega}(\vec{k} \times \vec{B}) = -\vec{E}$$

Simplifying this expression leads us to the desired result.

$$\frac{\omega}{k^2}(\vec{k} \times \vec{B}) = \boxed{\frac{v}{k}(\vec{k} \times \vec{B}) = -\vec{E}}$$

8 Creating an Electromagnetic Wave

A plane electromagnetic wave, of wavelength 3.0 m, travels in vacuum in the $+x$ direction. The electric field, of amplitude 300 V/m, oscillates parallel to the y axis.

1. Determine the Electric Field $\vec{E}(\vec{r}, t)$

   Three properties are required to determine $\vec{E}(\vec{r}, t)$: we need the amplitude of the wave $\vec{E_0}$, its wavenumber $\vec{k}$ and its angular frequency ω. We thus have

$$\|\vec{E_0}\| = 300 \frac{\mathrm{V}}{\mathrm{m}}, \hat{k} = \hat{i}, \|\vec{k}\| = \frac{2\pi}{\lambda} = \frac{2\pi}{3}\mathrm{m}^{-1}$$

$$\omega = 2\pi f \rightarrow f = \frac{v}{\lambda} = \frac{3 * 10^8}{3} = 10^8 \rightarrow \omega = 2\pi * 10^8 \mathrm{s}^{-1}$$

We now simply substitute all the above values to produce

$$\boxed{\vec{E}(\vec{r}, t) = (3\hat{j}) * 10^2 \cos\left(\frac{2\pi}{3}x - (2\pi * 10^8)t\right)}$$

2. Determine the Magnetic Field $\vec{B}(\vec{r}, t)$

   Recall that the Magnetic Field may be obtained by crossing the Electric Field $\vec{E}(\vec{r}, t)$ with the direction of propagation $\vec{k}$:

$$\vec{k} \times \vec{E} = \omega B \rightarrow \vec{B} = \frac{\|\vec{k}\|}{\omega}(\hat{s} \times \vec{E}) = \frac{1}{2\pi * 10^8} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} 10^2 \cos\left(\frac{2\pi}{3}x - (2\pi * 10^8)t\right)$$

$$\boxed{\vec{B}(\vec{r}, t) = \left[\frac{3}{2\pi}\hat{k}\right] * (10^{-6}) \cos\left(\frac{2\pi}{3}x - (2\pi * 10^8)t\right)}$$

3. What is the shortest distance along the wave between any two points that have a phase difference of 30°?

   Finding the distance along the wave requires an arc length integral from $0 \leq t \leq \frac{\pi}{6}$. Recall that the parametric arc length is given by

$$\int_0^{\frac{\pi}{6}} \sqrt{(E'_{0_x}(t))^2 + (E'_{0_y}(t))^2} dt$$

   As the wave oscillates along the y axis, there is no x component, which results in

$$\int_0^{\frac{\pi}{6}} -6\pi * 10^{10} \cos\left(\frac{2\pi}{3}x - 3\pi * 10^8 t\right) dt = \left. \frac{-6\pi * 10^{10} \sin(\frac{2\pi}{3}x - 3\pi * 10^8 t)}{-3\pi * 10^8} \right|_{t=0}^{t=\frac{\pi}{6}}$$

   This results in the following quite awkward – and thus most likely incorrect – expression

$$2 * 10^2 \left(\sin\left(\frac{2}{3}\pi x - 18\pi^2 * 10^8\right) - \sin\left(\frac{2}{3}\pi x\right)\right)$$

   Substituting the simplest x possible, $x = 0$, and taking the absolute value, we have[2]:

$$\boxed{d = 6.843302723}$$

[2]Yes, I'm also wondering about the units. Since this is a distance *along* a curve, I suspect the units would simply be meters, but I'm not sure.

4. What is the shortest time interval for a phase difference of 30° to occur at a fixed point along the wave?

   We may interpret this to ask: how long would we have to wait as the wave passed by for a 30° phase difference to develop? The answer, of course, depends on the wave's speed and period. Recall that $T = \frac{1}{f}$, which is the amount of seconds for the wave to complete one cycle. 30° is a sixth ($\frac{30}{360} = \frac{1}{6}$) of a full cycle, so it stands to reason that it would take $\frac{1}{6}T = \frac{1}{6f} = \boxed{\frac{1}{6 * 10^8}\ \text{s}}$.

5. What phase shift occurs at a given point in 10^{-6} s, and how many waves have passed by in that time?

   As the period $T = 10^{-8}$, we expect a $\frac{10^{-6}}{10^{-8}} * 2\pi = 200\pi = \boxed{0^\circ}$ phase shift, and $\frac{200\pi}{2\pi} = \boxed{100}$ waves to pass by.